Maturity transformation

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Long-term (fixed-rate) assets

Maturity transformation and liquidity risk is one of the main sources of uncertainty in the banks' balance sheet

We need to somehow capture the idea that the present value of a major proportion of loans will be sensitive to changes in short-term rates

Modelling multi-period loans literally would be very convenient:

- would require a larger number of new state variables
- Euler equation(s) on the household's side would become overly complex

Geometric loans

Instead, we introduce “geometric” loans:

- loans repaid infinitely long
- repayments geometrically decreasing (at a fixed rate)
- the “average maturity” parameterised using Macaulay’s duration

Because of the geometric design of the repayments, everything can be expressed recursively (both backward and dforward expressions)

Require only one new state variable

At time t, the household borrows a total amount of L_t.

From period $t+1$ on the household is supposed to repay

- $Q_t L_t$ in period $t+1$,
- $\psi \, Q_t L_t$ in period $t+2$,
- ...
- $\psi^{n-1} \, Q_t L_t$ in period $t+n$.
- ...

where Q_t is a "price" somehow related to implicit long-term interest rates.

Parameter ψ determines the "average maturity".

Loan demand

The household's budget constraint:

$$L_t = J_{t-1} + expenditure_t - income_t$$

- L_t is the new loan(s) taken today,
- J_{t-1} is the repayment of all existing loans

$$J_{t-1} := Q_{t-1}L_{t-1} + \psi\, Q_{t-2}L_{t-2} + \cdots + \psi^{n-1}\, Q_{t-n}L_{t-n} + \cdots$$

$$J_t = \psi J_{t-1} + Q_t L_t$$

FOC w.r.t. L_t

$$\Lambda_t = Q_t \mathsf{E}_t \left[\beta \Lambda_{t+1} + \psi \beta^2 \Lambda_{t+2} + \cdots \psi^{n-1} \beta^n \Lambda_{t+n} \ldots \right]$$

or recursively

$$1 = Q_t \, \Omega_t$$

$$\Omega_t = \mathsf{E}_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 + \psi \, \Omega_{t+1} \right) \right]$$

Loan supply

Competitive risk-neutral intermediaries transform short-term (one-period) funds to geometric loans.

The "price" of the geometric loan, Q_t, is determined so to make the present value of the repayments equal to the amount lent

$$1 = Q_t \mathsf{E}_t \left[\frac{1}{R_t} + \frac{\phi}{R_t R_{t+1}} + \cdots \frac{\phi^{n-1}}{R_t \cdots R_{t+n}} + \cdots \right]$$

using the short-term rate, R_t, and its future expectations, to discount future cash flows.

Put recursively

$$1 = Q_t \Psi_t$$

$$\Psi_t = \frac{1}{R_t} \mathsf{E}_t \left[1 + \phi \Psi_{t+1} \right]$$

Compare with the household's FOC $\Rightarrow \Omega_= \Psi_t$:

$$\mathsf{E}_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 + \psi\, \Omega_{t+1} \right) \right] = \frac{1}{R_t} \mathsf{E}_t \left[1 + \psi\, \Psi_{t+1} \right]$$

Note that up to first order

$$\frac{\beta \Lambda_{t+1}}{\Lambda_t} \approx \frac{1}{R_t}$$

In higher orders, a term premium will occur between the short-term rate and the houshold's pricing kernel, depending on the variance-covariance structure of the rest of the model.

Two main implications

1. The household's intertemporal choice (Euler equation) remains up to first order unchanged compared with the standard case when there are only one-period loans. **The intertemporal choice between C_t and C_{t+1} is still based on the underlying short-term rate**.

2. The first-order effects of the multi-period loans go only through **valuation**: When short-term rates increase **unexpectedly**, the present value of outstanding loans (loans made/taken in the past) fallse. The intermediaries run losses, the households gain.

Parameterising the maturity of the geometric loans

Macaulay's duration – a concept developed independently by several economists (between the 30s and 40s last century) to describe

- either the average maturity of unevenly distributed future cash flows
- or the elasticity of the prices of different types of assets w.r.t. changes in short-term rates.

Macaulay's duration

Present-value-weighted time to receive each cash flow. MD of the geometric loan:

$$d_t = 1 \times \frac{Q_t}{R_t} + 2 \times \frac{\psi\, Q_t}{R_t R_{t+1}} + \cdots + n \times \frac{\psi^{n-1}\, Q_t}{R_t \cdots R_{t+n-1}} + \cdots$$

In steady state (after subsituting away for Q_t):

$$d = \frac{R}{R - \psi} \quad \Rightarrow \quad \psi = R - R/d$$

Practical issues

Value of the outstanding loans for regulatory purposes: Mark-to-market versus book value

Timing of write-offs of losses occurring because of unexpected changes in short-term rates.