Financial accelerator: Contingent and non-contingent debt in DSGE models

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In this session. . .

Review BGG's financial accelerator; show its unsuitability for modelling the banking sector and macroprudential policies.

Modify the debt contract: make the lending rate non-contingent.

See how to code up the accelerators in DSGE models

Compute basic comparative statics to describe the properties of the accelerator

Show the differences between the two in simulations of a simple SOEMOLI model.

Why to study this?

We aim to examine the resilience/vulnerability of financial institutions to (all sorts of) macroeconomic risks.

BGG's financial accelerator uses state-contingent debt contracts:

- lending rates adjusted ex post in response to aggregate shocks,
- financial intermediarites run zero profits at all times,
- only the borrowers bear aggregate risk.

Fails to deliver realistic feedback between the real and financial sectors: the financial sector's balance sheet not exposed to aggregate risk.

Quick review of BGG's contract

Today:

- Enterpreneurs each with net worth N_t choose the amounts borrowed $L_t = P_{K,t}K_t - N_t$, based on the terms of the state-contingent debt contract.
- Banks are simple intermediaries shoveling funds from depositors (fixed risk-less rate, R_t) to investors-enterpreneurs.
- They specify a (standard debt) contract with different lending rates, $R_{L,t+1}$, for each possible future aggregate rate of return on capital, $R_{K,t+1}$.
- The contract terms always guarantee the banks receive $R_t\, L_t$ in $t+1$ whatever the aggregate productivity $R_{K,t+1}$.

Tomorrow:

- The aggregate rate of return, $R_{K,t+1}$ is observed. The corresponding lending rate, $R_{L,t+1}$, applies.

- Each enterpreneur observes his own idiosyncratic productivity, $\omega \sim F(\omega)$. Enterpreneur ω's total return is $R_{K,t+1} P_{K,t} K_t \, \omega$.

- Enterpreneurs who don't get sufficient return default:

$$R_{K,t+1} P_{K,t} K_t \omega < R_{L,t+1} L_t.$$

  This defines the cut-off produtivity level $\bar{\omega} := \frac{R_{L,t+1} L_t}{R_{K,t+1} P_{K,t} K_t}$.

- The others repay the loan, $R_{L,t+1} L_t \equiv R_{K,t+1} P_{K,t} K_t \, \bar{\omega}$, and keep the remainder of the their return as new net worth.

- Banks receive the repayments from the survived, and seize whatever the defaulted have produced less the monitoring cost.

- The total amount received by the banks is exactly $R_t\,L_t$ (the banks run zero profits). This is guaranteed by setting the lending rate, $R_{L,t+1}$, ex post sufficiently high to compensate for the defaults and the monitoring cost.

Who gets what?

Total return on capital $Z_t := R_{K,t}P_{K,t-1}K_{t-1}\underbrace{\int_0^\infty \omega f(\omega)d\omega}_{=1}$

The defaulted: $Z_t \int_0^{\bar{\omega}} \omega f(\omega)d\omega$

- Monitoring cost $\mu\, Z_t \int_0^{\bar{\omega}} \omega f(\omega)d\omega \rightarrow$ Social or private loss
- Recovered $(1-\mu)\, Z_t \int_0^{\bar{\omega}} \omega f(\omega)d\omega \rightarrow$ Banks

The survived: $Z_t \int_{\bar{\omega}}^\infty \omega f(\omega)d\omega$

- Repaid $Z_t\, \bar{\omega} \int_{\bar{\omega}}^\infty f(\omega)d\omega = R_{L,t}L_{t-1}\,[1-F(\bar{\omega})] \rightarrow$ Banks
- Retained $Z_t\,(1-\bar{\omega} \int_{\bar{\omega}}^\infty f(\omega)d\omega) \rightarrow$ Enterpreneurs

Modifying the debt contract

Make the lending rate fixed ex ante, not responding to shocks to the aggregate return on capital.

This looks simple, but has a far-reaching consequence for the design of the rest of the model:

- Whenever there's an unexpected aggregate shock, the banks will run losses or extra profits. . .
- . . . this means the banks must have their own net worth and/or access to equity markets to be able to absorb these.

We will happily ignore this fact at the moment assuming households supply (or receive) any excess or shortage of funds. Bank capital will be added to our framework later.

Formal optimisation problems

Max the expected profit of one agent (here: the enterpreneur) s.t. the other's participation constraint (here: the bank).

- BGG's state-contingent contract: Maximise over K_t and a continuum of $R_{L,t+1}$ (one for each possible $R_{K,t+1}$). The participation constraint is that the bank receives the risk-less return in every possible future (whatever $R_{K,t+1}$).

- The non-contingent contract: Maximise over K_t and a single $R_{L,t+1}$. The participation constraint is that the bank receives the risk-less return in expectations.*

*An important silent assumption is that both the enterpreneurs and the banks are risk-neutral.

The state-contingent contract

$$\max_{R_{L,t+1},K_t} \mathsf{E}_{R_{K,t+1}} \Big[R_{K,t+1} P_{K,t} K_t - \overbrace{R_{L,t+1} L_t \int_{\bar\omega}^{\infty} f(\omega) d\omega}^{\text{repayment if the enterpreneur survives}}$$

$$- \underbrace{R_{K,t+1} P_{K,t} K_t \int_0^{\bar\omega} \omega f(\omega) d\omega}_{\text{return seized by the bank if she defaults}} \Big]$$

subject to a continuum of constraints (for each $R_{K,t+1}$)

$$\underbrace{R_{L,t+1} L_t \int_{\bar\omega}^{\infty} f(\omega) d\omega}_{\text{repayment}} + \underbrace{(1-\mu) R_{K,t+1} P_{K,t} K_t \int_0^{\bar\omega} \omega f(\omega) d\omega}_{\text{recovered from the defaulted less monitoring}} = R_t L_t$$

with $L_t := P_{K,t} K_t - N_t$, and $\bar\omega := \frac{R_{L,t+1} L_t}{R_{K,t+1} P_{K,t} K_t}$.

The non-contingent contract

$$\max_{R_{L,t},K_t} \mathrm{E}_{R_{K,t+1}} \Bigg[R_{K,t+1} P_{K,t} K_t - \overbrace{R_{L,t} L_t \int_{\bar{\omega}}^{\infty} f(\omega) d\omega}^{\text{repayment if the enterpreneur survives}}$$

$$- \underbrace{R_{K,t+1} P_{K,t} K_t \int_0^{\bar{\omega}} \omega f(\omega) d\omega}_{\text{return seized by the bank if she defaults}} \Bigg]$$

subject to a single constraint

$$\mathrm{E}_{R_{K,t+1}} \Bigg[\underbrace{R_{L,t} L_t \int_{\bar{\omega}}^{\infty} f(\omega) d\omega}_{\text{repayment}} + \underbrace{(1-\mu) R_{K,t+1} P_{K,t} K_t \int_0^{\bar{\omega}} \omega f(\omega) d\omega}_{\text{defaults less monitoring}} \Bigg] = R_t L_t$$

with $L_t := P_{K,t} K_t - N_t$, and $\bar{\omega} := \frac{R_{L,t} L_t}{R_{K,t+1} P_{K,t} K_t}$.

How to code up the contracts?

There are a number of differences between the two contracts and their FOCs; with some of them very subtle (especially regarding the expectations). They arise only because of aggregate risk.

If interested in

- either "deterministic" simulations (i.e. future shocks are either foreseen as fixed numbers, or occur unexpectedly as deterministic disturbances)
- or first-order approximate simulations

(which is probably the case in more than 95 % of what practical modellers do) we can think of the solutions to the two contract problems as follows.

Timing matters!

1. Introduce auxiliary variables $R^*_{L,t}$, $\bar{\omega}^*_t$ (you'll also need one for the lagrange multiplier on the participation constraint).

2. Solve the optimal debt contract for these auxiliary variables and K_t at time t taking the lead ("expectations") of $R_{K,t+1}$ as a given fixed number (no aggregate uncertainty). With no aggregate uncertainty, the FOCs for the debt contract are simple (see e.g. BGG).

3. This step is to nail down K_t and L_t.

4. Now, move one period ahead, re-labeling the period in which the loan was made as $t-1$, and the current period in which the loan is supposed to be repaid as t.

5. The actual lending rate, $R_{L,t}$ and the actual cut-off, $\bar{\omega}_t$, will, in general, differ from the auxiliary ones ($R^*_{L,t-1}$, $\bar{\omega}^*_{t-1}$), depending on the actual $R_{K,t}$.

In the state-contingent world

6a. Use the banks' zero-profit condition to back out $\bar{\omega}_t$ and the definition of $\bar{\omega}_t$ to back out $R_{L,t}$.

$$R_{L,t}L_{t-1}\int_{\bar{\omega}}^{\infty} f(\omega)d\omega \;+\; (1-\mu)R_{K,t}P_{K,t-1}K_{t-1}\int_{0}^{\bar{\omega}}\omega f(\omega)d\omega = R_{t-1}L_{t-1}$$

$$\bar{\omega} = \frac{R_{L,t}L_{t-1}}{R_{K,t}P_{K,t-1}K_{t-1}}.$$

The integrals look frighteningly, but they're in fact friendly – see BGG.

In the non-contingent world

6b. Set the actual lending rate equal to the auxiliary one determined in the previous period,

$$R_{L,t} := R^*_{L,t-1}.$$

Use the definition of $\bar{\omega}_t$

$$\bar{\omega} = \frac{R_{L,t} L_t}{R_{K,t} P_{K,t-1} K_{t-1}}.$$

Note the zero-profit condition will not hold in general in this case.