Simulating asset price bubbles

Economic Modelling Unit, IMF
jbenes@imf.org

April 2010

In this session. . .

Define rational, irrational and magic bubbles

See how to code up the bubbles in DSGE models

Simulate a capital price bubble

Why to study this?

Asset prices are one of the key drivers in the credit channel of monetary (and macroprudential) policy.

They determine the value against which the agents can borrow– e.g. net worth of enterpreneurs in the accelerator model (e.g. Bernanke, Gertler, Gilchrist, 1999), or house prices in collateral models (e.g. Iacoviello, 2005).

Hence, the asset prices affect the effective costs of borrowing (the market premium, or the shadow value of borrowed funds), and aggregate demand.

Excessive asset price growth can amount to a build-up of risk on the balance sheets of financial institutions.

Types of asset price bubbles

Bubbles are deviations of the observed (market) prices from the fundamental price of the underlying asset.

Rational bubbles (e.g. Blanchard, Watson, 1982):

- a non-fundametal solution of the arbitrage condition, with no arbitrage opportunities (unstable node),
- require modifications in the model solution technique.

Irrational bubbles (e.g. Bernanke, Gertler, 1999):

- exogenous deviations from the arbitrage condition itself,
- can be simulated using the common techniques.

Magic bubbles:

- the asset price follows its fundamental path,
- however, expectations driven high by overestimating future productivity/performance that never materialises,
- can be simulated using the common techniques (provided you can combine expected and unexpected shocks).

More technical details

A typical arbitrage condition for an asset price looks like this:

$$P_t = \frac{1}{R_t} \mathsf{E}_t \left[\underbrace{Q_{t+1}}_{\text{dividents,rents,etc.}} + \underbrace{(1-\delta)P_{t+1}}_{\text{capital gain less depreciation (if any)}} \right].$$

Iterated forward, the equation has infinitely many solutions (assuming Q_t is well-behaved):

- one solution is stable, with $\lim_{k\to\infty} \mathsf{E}_t \left[\frac{1}{R_t \cdots R_{t+k-1}} P_{t+k} \right] = 0$,
- all other solutions are unstable.

Bubble simulations:

- No-bubble simulations put the asset prices on the single (unique) stable node.
- Rational-bubble simulations put the asset prices on one of the unstable nodes.
- Irrational-bubble simulations break the arbitrage condition.

(Slightly modified) Bernanke-Gertler irrational bubbles

Denote

- the fundamental price by $\tilde{P}_t$ ($\leftarrow$the stable solution of the arbitrage condition),
- the observed market price by P_t.

A bubble, b_t, is a log discrepancy between the two

$$b_t := \log P_t - \log \tilde{P}_t$$

Once arisen, a bubble is expected

- to explosively growth at a rate $\phi > 1$ into next period with probability $1 - \tau$,
- to burst in next period with probability τ.

The $t + 1$ expected value is therefore

$$\mathsf{E}_t[b_{t+1}] = \tau \times 0 + (1 - \tau) \times \phi\, b_t = (1 - \tau)\, \phi\, b_t$$

At this point we assume $(1-\tau)\times\phi<1$:

- While existing, the bubble is gradually exploding,
- but the conditional expectations are stable at all times.

Note also that whenever a positive bubble exists:

- the expected return on the asset is always smaller than the fundamental return

$$\mathsf{E}_t\left[Q_{t+1}+(1-\delta)P_{t+1}\right]<\mathsf{E}_t\left[Q_{t+1}+(1-\delta)\tilde{P}_{t+1}\right],$$

- the ex-post actual return is always greater than the expected return

$$\mathsf{E}_t\left[Q_{t+1}+(1-\delta)P_{t+1}\right]<Q_{t+1}+(1-\delta)P_{t+1},$$

How to code up irrational bubbles

We need to simulate a process b_t that is temporarily exploding but whose expectations are stable.

Introduce an autoregressive process

$$b_t = \rho_b b_{t-1} + \epsilon_{b,t}, \quad \text{where } \rho_b := (1 - \tau)\,\phi.$$

To simulate a bubble, use a sequence of unexpected $\epsilon_{b,t}$'s to drive the bubble along the desired explosive path. Because the shocks are unexpected, the expectatons will always be determined by $\mathsf{E}_t[b_{t+1}] = \rho_b\, b_t$.