Are the Responses of the U.S. Economy Asymmetric in Energy Price Increases and Decreases?

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The Literature on Oil Prices and the U.S. Economy

● Since the 1970s oil price shocks have been one of the leading candidates for explaining U.S. postwar recessions.

<u>Problem</u>: Finding an oil price shock measure that "works" in a VAR context is not straightforward (BGW 1997):

→ "anomalous" macroeconomic outcomes relative to the conventional wisdom.
→ "unstable" relationship with macroeconomic outcomes, as the sample is lengthened.

● Search for increasingly complicated specifications of the "true" relationship between oil prices and the U.S. economy, when linear and symmetric models failed to produce the desired results.

Asymmetric Models of the Transmission of Oil Price Shocks

Censored regression models seem to deliver larger and more stable responses:

- 1st Generation (Mork 1989)

Oil price increase:

$$\Delta p_t^+ = \begin{cases} \Delta p_t & if \ \Delta p_t > 0 \\ 0 & if \ \Delta p_t \leq 0 \end{cases}$$

- 2nd Generation (Hamilton 1996, 2003)

Net oil price increase:

$$\Delta p_t^{+,net} = \max[0, p_t - p_t^*],$$

where p_t^* is the highest oil price in the preceding 12 (or 36) months.

Two Types of Studies in the Literature

1. Studies that test for symmetry based on regression slopes (e.g., Mork JPE 1989, Hooker JMCB 2002, Balke et al. EJ 2002, Hamilton JoE 2003).

2. Studies that estimate the dynamic response of macroeconomic aggregates to percent increases or net percent increases in the price of oil. Examples:

Hooker (JME 1996); Bernanke, Gertler and Watson (BPEA 1997); Davis and Haltiwanger (JME 2001); Lee and Ni (JME 2002); Hamilton (JoE 2003); Leduc and Sill (JME 2004); Hamilton and Herrera (JMCB 2004).

Prototypical recursively identified VAR model:

$$\begin{pmatrix} \Delta p_t^{+} \\ \Delta gdp_t \end{pmatrix} \sim VAR(p) \text{ or } \begin{pmatrix} \Delta p_t^{+,net} \\ \Delta gdp_t \end{pmatrix} \sim VAR(p)$$

Consensus in the Macroeconomic Literature

The "evidence for asymmetric responses to oil price ups and downs is well established" (Davis and Haltiwanger 2001).

Why?

- Tests on slope parameters sometimes reject symmetry.

- More stable results?

- Censoring produces "better looking" VAR impulse responses (BGW 1997).

- Nicely complements some theoretical models of the transmission of oil price shocks (Bernanke 1983; Hamilton 1988; Pindyck 1991). We need these models to explain large effects of energy price shocks.

Limitations of Existing Estimates of Asymmetric Responses from Censored VAR Models

1. Censored oil price VAR models are fundamentally misspecified because the asymmetric DGP in question cannot be represented as a VAR model of any kind. As a result, the parameter estimates are inconsistent.

2. The implied impulse responses have been computed incorrectly.

3. Whether asymmetric responses differ from responses based on linear symmetric models has not been tested properly. Existing tests are inadequate for this purpose.

Stylized Static Model: Symmetric Case

<u>DGP:</u>

$$\begin{aligned} x_t &= \alpha_1 + \varepsilon_{1,t} \\ y_t &= \alpha_2 + x_t\beta + \varepsilon_{2,t} \end{aligned} \tag{1}$$

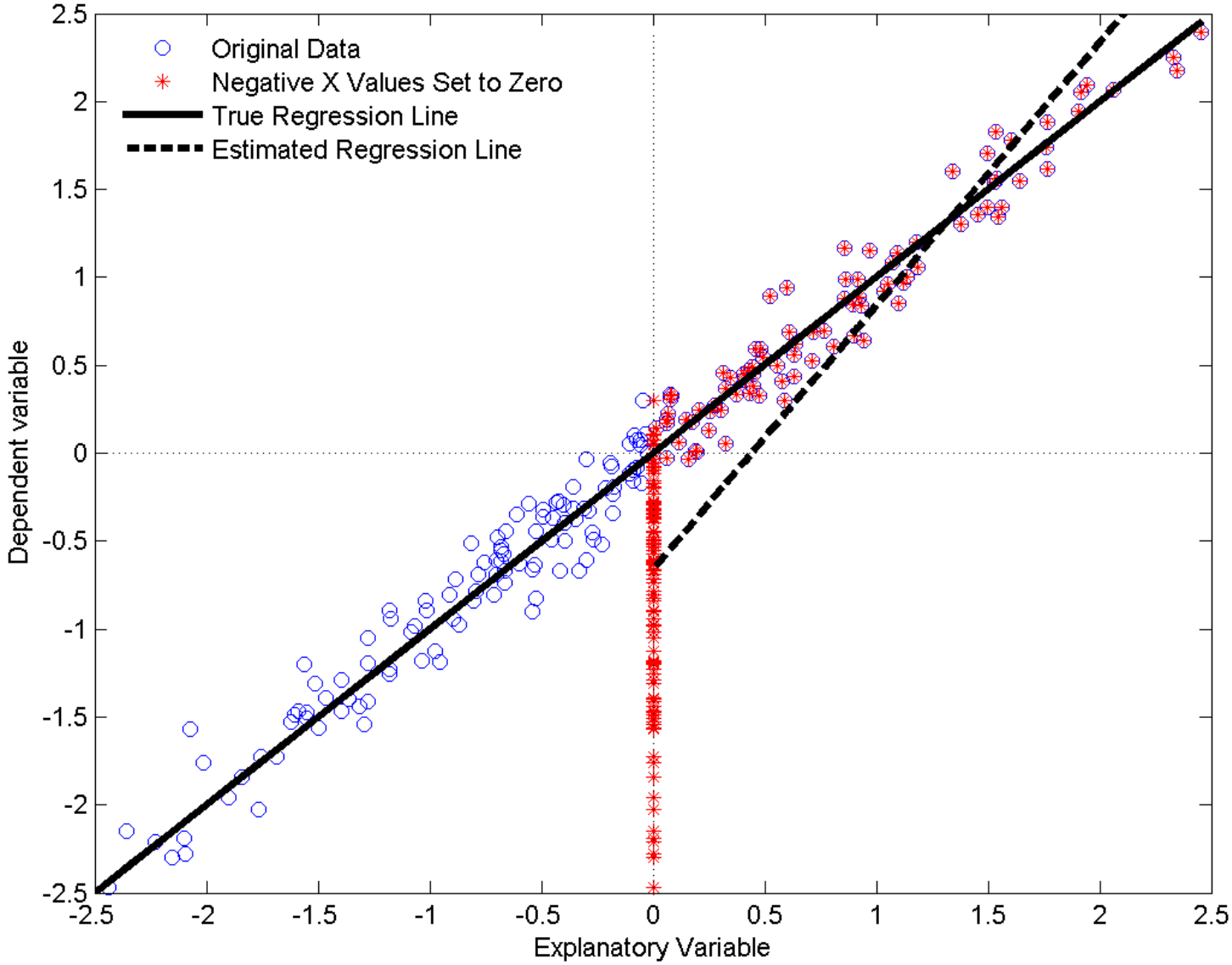
where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are mean zero iid Gaussian with variances σ_1^2 and σ_2^2.

<u>Censored Regressor Model:</u>

$y_t = a + x_t^+ b + u_t$ where $x_t^+ = \begin{cases} x_t & if \;\; x_t > 0 \\ 0 & if \;\; x_t \le 0 \end{cases}$ $\Rightarrow \hat{b}$ is inconsistent for β

(upward bias)

The Effect of Censoring Negative Values of the Regressor



How Big Is the Asymptotic Bias?

- Suppose $\alpha = 0$ and x_t has a symmetric distribution with mean zero and variance 1. Also suppose $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are uncorrelated. Then:

$$\hat{b} \xrightarrow{p} -\beta \frac{0.5\mu}{1-0.4\mu^2} \text{ where } \mu \equiv E(x_t \mid x_t > 0).$$

- If $x_t \sim NID(0,1)$, we obtain:

$$\hat{b} \xrightarrow{p} 1.47\beta$$

Stylized Static Model: Asymmetric Case

<u>DGP:</u>

$$\begin{aligned} x_t &= \alpha_1 + \varepsilon_{1,t} \\ y_t &= \alpha_2 + x_t\beta + x_t^+\gamma + \varepsilon_{2,t} \end{aligned} \tag{2}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are mean zero iid Gaussian with variances σ_1^2 and σ_2^2.

<u>Censored Regressor Model:</u>

$y_t = a + x_t^+ b + u_t$ $\Rightarrow \hat{b}$ is inconsistent for $\beta + \gamma$ unless $\beta = 0$

(upward bias)

Extensions to Dynamic Models

Standard approach:

$$\begin{pmatrix} x_t^+ \\ y_t \end{pmatrix} \sim VAR(p)$$

BGW (BPEA 1997, p. 103):

"Mork provided evidence that only positive changes in the relative price of oil have important effects on output. Accordingly, in our VARs we employ an indicator that equals the log-difference of the relative price of oil when that change is positive and otherwise is zero."

Leduc & Sill (JME 2004, p. 790):

"To get an empirical estimate of the output response to positive oil-price shocks, we run a VAR using … oil-price increases […] constructed by taking the first difference of the log of oil prices, then setting negative values to zero. Thus, only oil-price increases affect the other variables in the system."

What if the DGP is a Linear Symmetric VAR?

- DGP: Linear symmetric VAR(6) fitted on actual data for 1973.1-2007.12.

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} \sim VAR(6)$$

- Regression model: Recursively identified censored VAR

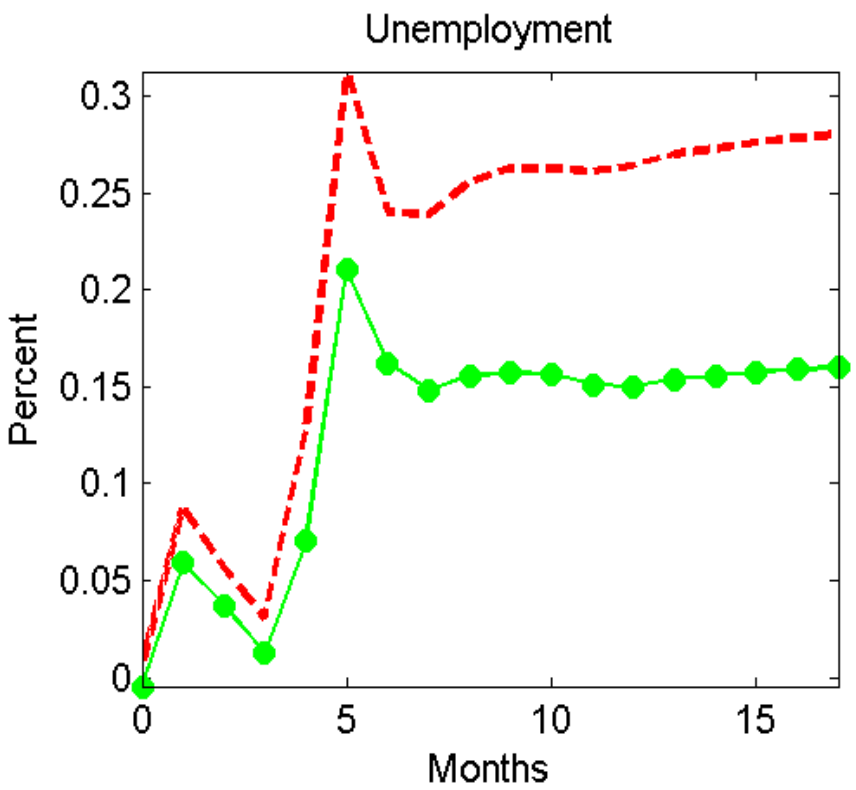
$$\begin{pmatrix} x_t^+ \\ y_t \end{pmatrix} \sim VAR(6)$$

where x_t and y_t are expressed in growth rates and y_t denotes the macroeconomic aggregate of interest.

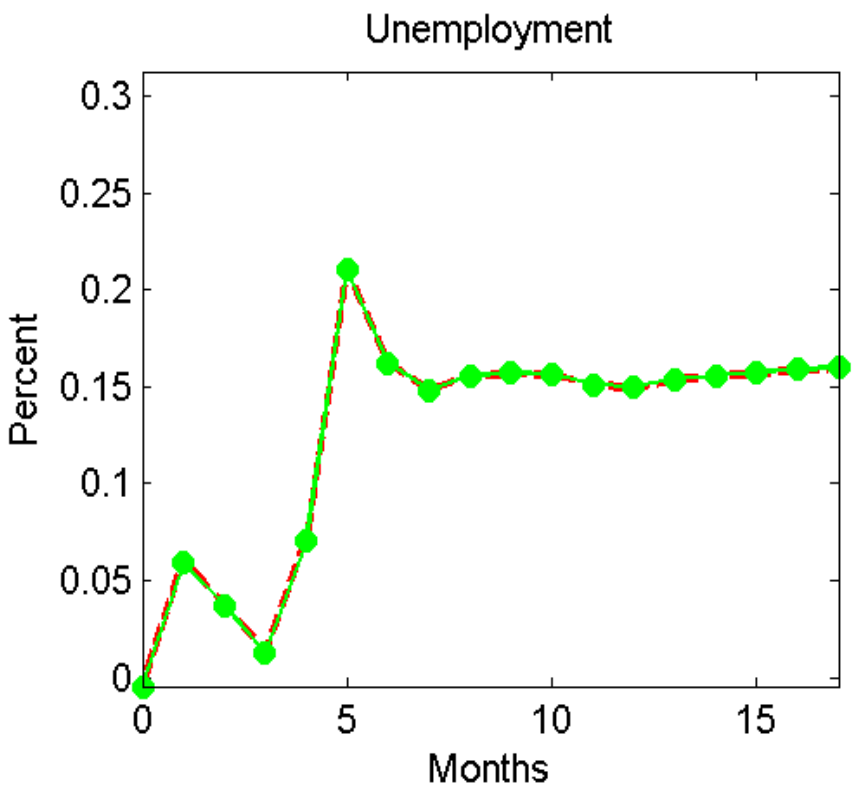
<u>Remark</u>: In the empirical literature, impulse responses from this model have routinely been computed exactly as in linear VAR models. For now, we will follow that practice, so we can assess the empirical results in the literature.

Inconsistency of the Estimated Effect of Energy Price Increases
Symmetric VAR DGP

Censored VAR



Symmetric VAR



Notes: T=100,000.

What if the DGP is an Asymmetric Dynamic Model?

● A censored VAR will generate realizations $x_t^+ < 0$ with positive probability.

The source of this problem is that the regression model is an incomplete description of the DGP.

● It is tempting to deal with this problem by censoring realizations with the wrong sign. In that case, the same type of asymptotic bias from censoring arises, as for the linear symmetric VAR-DGP.

Proposal for a Fully Specified Asymmetric DGP

- Strictly asymmetric structural DGP:

$$
\begin{aligned}
x_t &= \sum_{i=1}^{p} b_{11,i} x_{t-i} + \sum_{i=1}^{p} b_{12,i} y_{t-i} + \varepsilon_{1,t} \\
y_t &= \sum_{i=0}^{p} b_{21,i} x_{t-i}^{+} + \sum_{i=1}^{p} b_{22,i} y_{t-i} + \varepsilon_{2,t}
\end{aligned}
\tag{3}
$$

where the structural shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are uncorrelated. Although the slope parameters can be estimated consistently by OLS, the resulting residuals will not be uncorrelated. To impose that restriction, we need a restricted MLE.

- Regression:

$$
\begin{aligned}
x_t^{+} &= \sum_{i=1}^{p} b_{11,i} x_{t-i}^{+} + \sum_{i=1}^{p} b_{12,i} y_{t-i} + \varepsilon_{1,t} \\
y_t &= \sum_{i=1}^{p} b_{21,i} x_{t-i}^{+} + \sum_{i=1}^{p} b_{22,i} y_{t-i} + \varepsilon_{2,t}
\end{aligned}
$$

Unlike in the static model, the censored VAR will be inconsistent even if x_t has zero coefficients $\forall t$ in the second equation of the DGP!

A General Model of the Oil Price-Economy Link

● If we do not know whether there is an asymmetry or what form it takes, we face a dilemma. Which model should we fit to the data?

● Proposal:

$$
\begin{aligned}
x_t &= \sum_{i=1}^{p} b_{11,i} x_{t-i} + \sum_{i=1}^{p} b_{12,i} y_{t-i} + \varepsilon_{1,t} \\
y_t &= \sum_{i=0}^{p} b_{21,i} x_{t-i} + \sum_{i=1}^{p} b_{22,i} y_{t-i} + \sum_{i=0}^{p} g_{21,i} x_{t-i}^{+} + \varepsilon_{2,t}
\end{aligned}
\tag{4}
$$

1. Consistent estimation of each equation by OLS is possible.

2. This model eliminates the inconsistency (at the price of being possibly inefficient).

Computing Asymmetric Impulse Responses Properly

● The standard approach to constructing impulse responses is misleading since it ignores the path dependence of the future values of y_t and their dependence on the magnitude of the shock.

● The computation of impulse responses for nonlinear multivariate reduced form models is discussed in Koop et al. (1996).

<u>Problem</u>: 1. Shock is not uniquely defined.
2. Experiment violates the ceteris paribus assumption.

<u>Solution</u>: In the context of our structural asymmetric model, in contrast, the errors are mutually uncorrelated, so we can compute economically meaningful responses by drawing from the marginal distribution of structural shocks.

Computing Nonlinear Responses to Unanticipated Energy Price Increases

Having estimated the asymmetric model as discussed earlier, proceed as follows:

Step 1: Take a block of p consecutive values of x_t and y_t. This defines a history Ω^i.

Step 2. Given Ω^i, simulate two time paths for x_{t+i} and y_{t+i}, $i = 0,1,...,h$. In generating the first time path, the value of $\varepsilon_{1,t}$ is equal to a prespecified value δ. In generating the other time path, the value of $\varepsilon_{1,t}$ is drawn from the marginal empirical distribution of $\varepsilon_{1,t}$. The values of all subsequent shocks $\varepsilon_{1,t+i}$ and the value of $\varepsilon_{2,t+i}$, $i = 0,1,2,...,h$, are drawn from their respective marginal distributions. In practice, we treat these draws as independent.

Step 3: Calculate the difference between the time paths for y_{t+i}, $i = 0,1,...,h$.

Step 4: Average this difference across *m=10,000* repetitions of Steps 2 and 3.

- This average is the response of y_{t+i} to a shock of magnitude δ <u>conditional</u> on Ω^i :

$$I_y(h,\delta,\Omega^i)$$

is the relevant statistic for forecasting and policy work.

- The corresponding <u>unconditional</u> response

$$I_y(h,\delta) = \int I_y(h,\delta,\Omega^i)d\Omega^i$$

is a measure of the general of the general importance of oil price shocks.

● The response typically used in existing empirical work is

$$I_y^*(h, \delta, \underline{0})$$

where we condition on a <u>hypothetical</u> initial condition, in which all variables equal zero, and substitute expected values of future shocks rather than integrating over all possible shock paths.

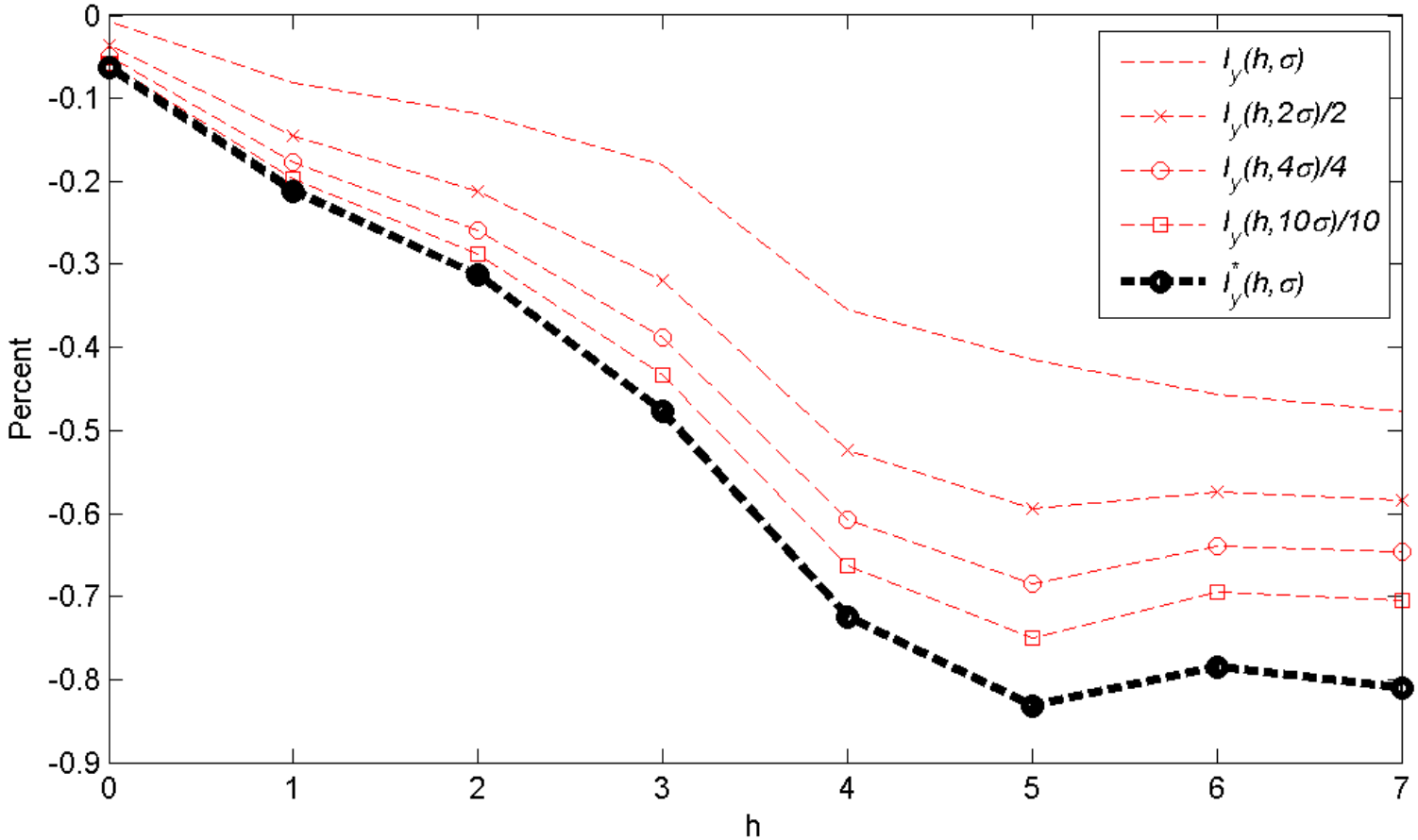
<u>Problems:</u>

1. Koop et al. (1996) show that this traditional impulse response may not converge to zero, even when the stochastic process is stationary.

2. Potter (2000): Theoretical treatment of nonlinear impulse responses is facilitated by treating future shocks as random variables rather than fixed values.

3. Why is this specific history interesting?

How Different is the Traditional Response from the Correctly Computed Unconditional Response?



Notes: Response of GDP to a positive oil price shock. $I_y(h,\sigma)$ is computed by Monte Carlo integration over 300 histories with 10,000 paths each. The responses have been scaled for compatibility. h denotes the horizon.

Testing for Symmetry in the Impulse Responses

● The <u>traditional approach</u> to testing for symmetry in the transmission of energy price shocks involves tests on the symmetry of slope coefficients in regressions of y_t on lagged x_t^+ and x_t^- (e.g., Mork 1989).

This is equivalent to testing $H_0 : g_{21,1} = ... = g_{21,p} = 0$ in the structural model.

● The structural model suggests that Mork's (1989) reduced form test fails to impose all restrictions. A test of all symmetry restrictions on the slopes involves:

$$H_0 : g_{21,0} = ... = g_{21,p} = 0$$

Empirical Results: Slope-Based Symmetry Tests
Baseline Model with 6 Lags

Variable	The Proposed Test of Symmetric Slope Coefficients	Marginal Significance Level	Mork's Test of Symmetric Slope Coefficients	Marginal Significance Level
Unemployment	7.722	0.358	3.132	0.792
Gasoline Consumption	11.376	0.123	9.237	0.161
Real GDP	10.472	0.163	9.757	0.135

Limitations of Slope-Based Symmetry Tests

1. Given the nonlinearity of the impulse response in the reduced form parameters, even 1% rejection of symmetry based on slopes does not guarantee large degree of asymmetry in the impulse responses.

Likewise, statistically insignificant departures from symmetric slopes may translate to a large and/or statistically significant degree of asymmetry in impulse responses.

2. Slope-based tests ignore that the extent to which the responses in the symmetric linear model provide a good approximation depends on the magnitude of the shock we consider.

⇨ This suggests that we test symmetry directly on the object of interest.

A New Test of the Symmetry of Impulse Responses

● Estimate the general (potentially nonlinear) model and compute the impulse responses to a positive and a negative energy price shock, as discussed earlier.

● Construct a Wald test of $H_0 : I_y(h,\delta) = -I_y(h,-\delta)$ for $h = 0,....,H$.

● The test statistic has an asymptotic χ^2_{H+1} distribution.

<u>Remarks:</u>

The variance-covariance matrix of the vector sum of response coefficients can be estimated by bootstrap simulation.

Size of the 5% Test of Symmetric Responses

	Unemployment		Gas Consumption		Real GDP	
H	1 std dev	2 std dev	1 std dev	2 std dev	1 std dev	2 std dev
2	0.06	0.05	0.06	0.05	0.06	0.05
4	0.07	0.05	0.07	0.05	0.07	0.05
6	0.08	0.05	0.08	0.06	0.07	0.05
8	0.09	0.07	0.10	0.07	0.09	0.07

Notes: Based on average of 90 histories and 20,000 draws under the null.

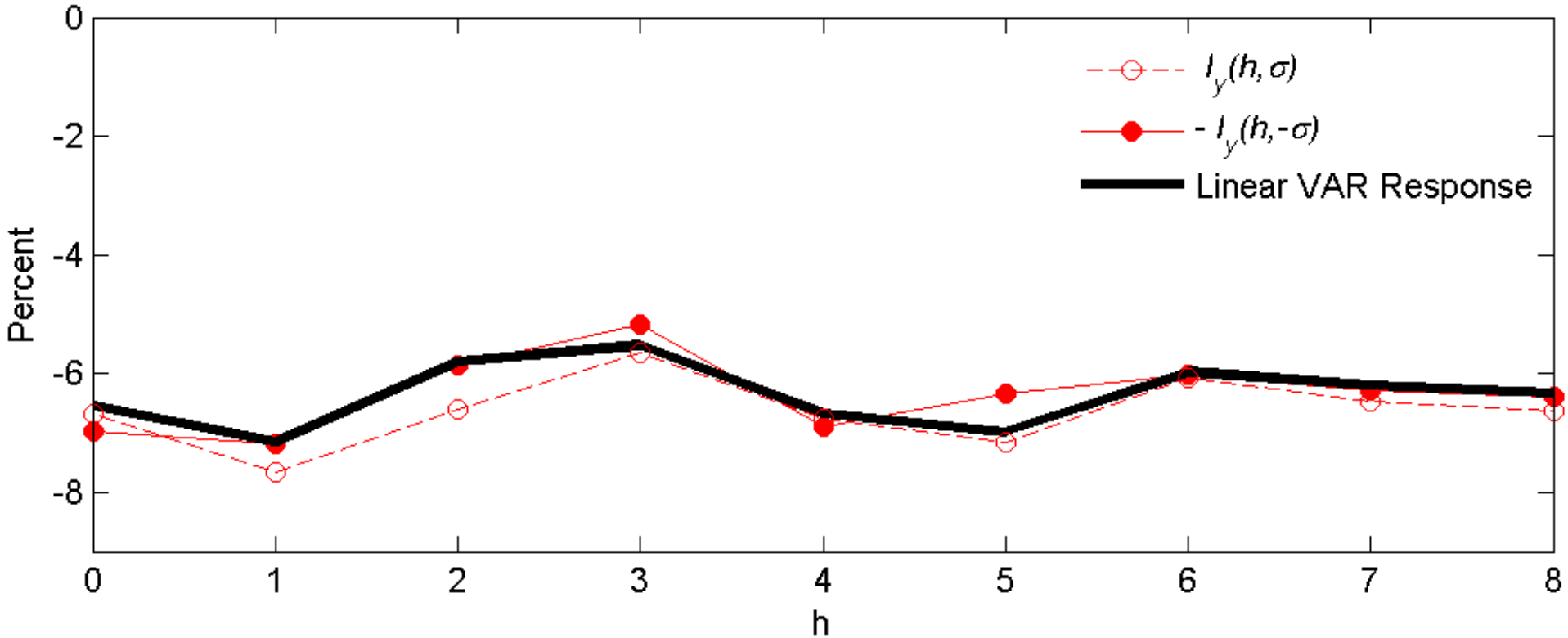
Empirical Results of Tests for Symmetric Responses

p-Values for Test of $H_0 : I_y(h,\delta) = -I_y(h,-\delta)$ for $h = 0,....,H$.

H	Gas Consumption 1 Std. Deviation Shock	Gas Consumption 2 Std. Deviation Shock	GDP 1 Std. Deviation Shock	GDP 2 Std. Deviation Shock	Unemployment 1 Std. Deviation Shock	Unemployment 2 Std. Deviation Shock
0	0.45	0.47	0.40	0.47	0.36	0.43
1	0.13	0.28	0.44	0.54	0.65	0.73
2	0.05	0.15	0.59	0.69	0.83	0.88
3	0.09	0.25	0.56	0.68	0.92	0.96
4	0.07	0.21	0.66	0.78	0.97	0.98
5	**0.04**	0.15	0.78	0.87	0.99	1.00
6	0.06	0.18	0.48	0.59	1.00	1.00
7	0.09	0.26	0.58	0.69	1.00	1.00

Notes: Based on 20,000 simulations of Model (4). p-values based on the χ^2_{H+1}-distribution.

Gasoline Consumption Example:
How Different are the Response Estimates?
Baseline Model with 6 Lags



Testing Models of Net Energy Price Increases

● Existing tests of the net increase model are based on one-step ahead single-equation predictive models:

$$y_t = \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{i=1}^{p} \gamma_i x_{t-i} + \sum_{i=1}^{p} \delta_i x_{t-i}^{+,net} + u_t \quad \text{Balke et al. (2002)}$$

$$y_t = \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{i=1}^{p} \delta_i x_{t-i}^{+,net} + u_t \quad \text{Hamilton (2003)}$$

⇨ What are the implications for degree of asymmetry of impulse responses?

⇨ Existing evidence against the linear symmetric VAR model does <u>not</u> justify the model:

$$\begin{pmatrix} x_t^{+,net} \\ y_t \end{pmatrix} \sim VAR(p)$$

That model is misspecified and its impulse responses have been routinely computed incorrectly (also see Balke et al. 2002).

Testing Symmetry in Models of Net Energy Price Increases

$$
\begin{aligned}
x_t &= \sum_{i=1}^{p} b_{11,i} x_{t-i} + \sum_{i=1}^{p} b_{12,i} y_{t-i} + \varepsilon_{1,t} \\
y_t &= \sum_{i=0}^{p} b_{21,i} x_{t-i} + \sum_{i=1}^{p} b_{22,i} y_{t-i} + \sum_{i=0}^{p} g_{21,i} x_{t-i}^{+,net} + \varepsilon_{2,t}
\end{aligned}
\tag{5}
$$

Slope-based test: $H_0 : g_{21,0} = ... = g_{21,p} = 0$

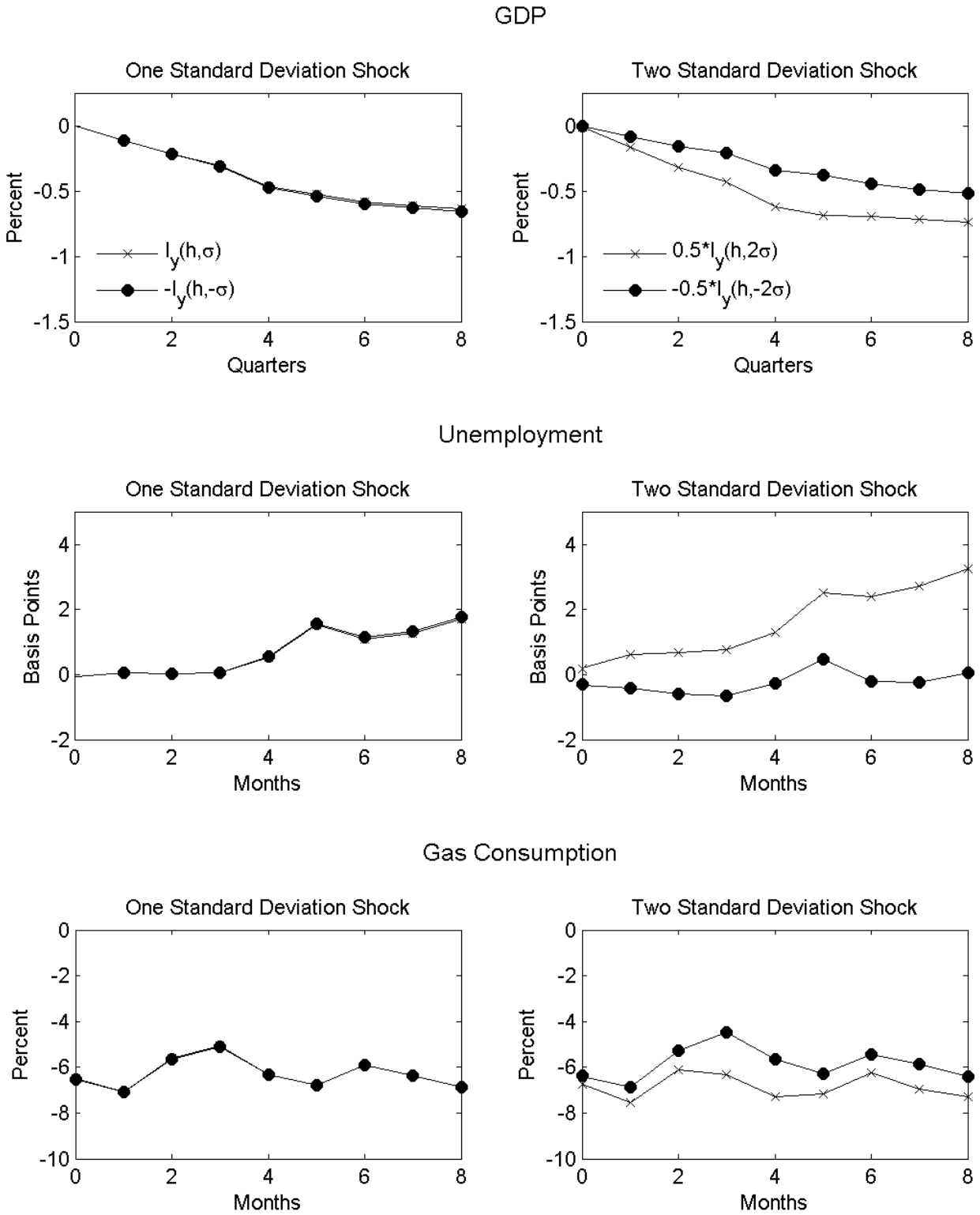
Impulse-response based test: $H_0 :\ I_y(h,\delta) = -I_y(h,-\delta)$ for $h = 0,....,H$.

Slope-Based Test of the Linear Symmetric VAR Model against the 3-Year Net Increase VAR Model

Variable	3-Year Net Increase	
	Test of Linear Symmetric Model	Marginal Significance Level
Unemployment	9.6332	0.210
Gasoline Consumption	14.5307	**0.043**
Real GDP	14.2965	**0.046**

NOTES: *p*-values based on χ^2_{H+1} distribution.

Responses to Energy Price Shocks
3-Year Net Increase Model



p-Values of Tests of $H_0 : I_y(h,\delta) = -I_y(h,-\delta)$ for $h = 0,....,H$

3-Year Net Increase Model

H	Gas Consumption 1 Std. Deviation Shock	Gas Consumption 2 Std. Deviation Shock	GDP 1 Std. Deviation Shock	GDP 2 Std. Deviation Shock	Unemployment 1 Std. Deviation Shock	Unemployment 2 Std. Deviation Shock
0	0.94	0.43	0.98	0.79	0.95	0.13
1	0.98	0.56	0.99	0.30	0.99	0.24
2	1.00	0.66	1.00	0.26	1.00	0.40
3	1.00	0.23	1.00	0.38	1.00	0.57
4	1.00	0.32	1.00	0.41	1.00	0.71
5	1.00	0.44	1.00	0.53	1.00	0.69
6	1.00	0.55	1.00	0.57	1.00	0.69
7	0.94	0.43	0.98	0.79	0.95	0.13

NOTES: p-Values based on χ^2_{H+1} distribution. Bootstrap variance estimates using 20,000 replications of model (5).

Implications for the Literature on the Transmission of Oil Price Shocks

● Theoretical models of sectoral reallocations or of delayed investment are <u>inconsistent</u> with our test results.

These are precisely the models required to explain potentially large effects of oil price shocks on U.S. output.

● In contrast, traditional models of cost-push and aggregate demand reduction are <u>consistent</u> with the lack of asymmetry.

These models do not predict large fluctuations in U.S. output in response to oil price shocks, consistent with the evidence from linear symmetric models.

Conclusions

1. Asymmetric responses should be estimated and tested using the econometric methods outlined in this paper. Earlier methods are invalid.

2. There is no statistically significant evidence of asymmetric responses to energy price shocks:

→ There is no compelling reason to abandon the use of linear symmetric models in empirical work on the transmission of energy price shocks (or the use of linear approximations to the steady state in theoretical work).

→ Theoretical models of the transmission of energy price shocks that imply asymmetries are not consistent with the U.S. data.

3. Key empirical studies that have shaped our thinking about how monetary policy has responded to oil price shocks are invalid.

Much of the evidence on plant level and sectoral effects of oil price shocks will also have to be reexamined.