MACRO-LINKAGES, OIL PRICES AND DEFLATION WORKSHOP
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The Global Integrated Monetary and Fiscal Model (GIMF)

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1 Introduction

Evolution of Multicountry Models at the Fund and Elsewhere

- 1980s-1990s: Models with partial microfoundations - MULTIMOD (IMF), FRBUS (Fed), etc.

- 1990s-today: Open economy **monetary** business cycle models - GEM (IMF), SIGMA (Fed), etc.

- 2007: Introduction of **fiscal** features in the new GIMF.

- 2008: Introduction of **macro-financial linkages** (financial accelerator) in the new GIMF.

Monetary Policy in Current Multicountry Models

- Monetary policy is the center of attention.
- Nominal rigidities and monetary policy reaction functions capture the interaction between monetary policy and the real economy fairly well.
- Real rigidities add realism and improve empirical performance: Habit persistence, investment adjustment costs, variable capital utilization, import adjustment costs.
- Problem: Almost complete absence of
  - Fiscal transmission channels.
  - Financial transmission channels.

Fiscal Policy in Current Multicountry Models

- Little work, except (initially) **balanced budget rules**.
- Typically Ricardian, except (recently) **liquidity constrained agents**.
- Typically government spending is **wasteful**, which is much too simple.
- Problem: Difficulties in replicating dynamic effects of fiscal policy.

Macro-Financial Linkages in Current Multicountry Models

- Only little work on **bank intermediation** has entered mainstream models (e.g. Christiano et al.).

- Some work on the **housing credit channel** (e.g. Iacoviello).

- Most of the work on the **corporate credit channel** (financial accelerator, Bernanke, Gertler and Gilchrist) has so far been applied to developing countries (Curdia, Bernanke/Gertler/Natalucci, Elekdag/Tchakarov, Devereux/Lane/Xu). This is also true for Mendoza's work on occasionally binding financial constraints. The exception is Christiano/Motto/Rostagno.

- Problem: Difficulties in replicating the critical interactions between the financial sector and the real economy.

GIMF = Global Integrated Monetary and Fiscal Model

- Equipped for Monetary Policy Analysis:
  - Nominal and real rigidities.
  - Monetary policy reaction function.
- Equipped for Fiscal Policy Analysis:
  - Multiple and powerful non-Ricardian features.
  - Fiscal policy reaction function.
- Equipped for Macro-Financial Linkages:
  - Corporate credit channel: Financial accelerator.
  - Bank intermediation: Represented by
    1. Endogenous external finance premium.
    2. Exogenous spread between government and corporate interest rates.

Fiscal Policy in GIMF: Four Reasons for the Breakdown of Ricardian Equivalence

(in INCREASING order of importance)

1. Multiple (three) distortionary taxes.

2. LIQ (liquidity constrained) agents without access to financial markets.

3. Lifecycle income patterns - wealth less dependent on future labor income.

4. OLG agents with finite lifetimes - high subjective discount rates.

Government Spending in GIMF

1. Shares of wasteful (or alternatively utility generating) and productive (output generating) government spending can be calibrated.

2. Government investment adds to a public capital stock $\Longrightarrow$ affects the productivity of private factors of production.

Other Features of GIMF

- Household Preferences:
  1. Households are myopic or liquidity constrained.
  2. General CRRA (not log): Intertemporal EoS is critical.
  3. Endogenous labor supply.

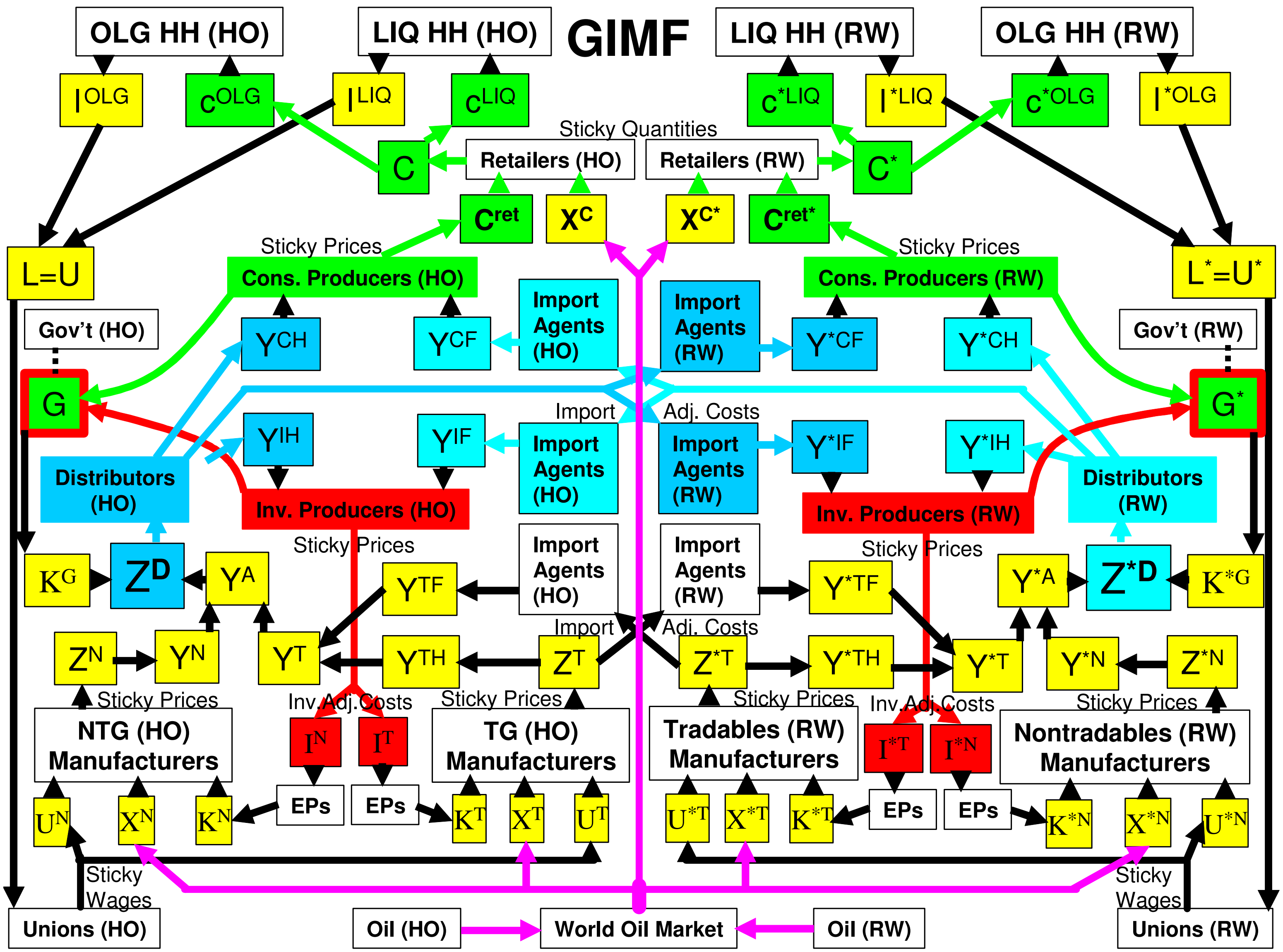
- Firm Technology:
  1. Firms are owned by OLG households and are therefore myopic.
  2. Traded and nontraded goods.
  3. Endogenous capital formation.
  4. Idiosyncratic productivity risks plus costly state verification require an external finance premium.

Other Features of GIMF (continued)

- Nominal Rigidities:
  1. Multiple (cascading) price rigidities.
  2. Nominal (or real) wage rigidity.
  3. Pricing to market.

- Real Rigidities:
  1. Consumption: Habit persistence (external) and retail quantity adjustment costs.
  2. Investment: Investment adjustment costs and variable capital utilization.
  3. Trade: Import adjustment costs.
  4. Oil input adjustment costs.

2 Model Overview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3 OLG Households

Births and Deaths

- Births each period: $Nn^t\left(1-\frac{\theta}{n}\right)$.
- Constant probability of death $(1-\theta) \Longrightarrow$ average economic lifetime is $1/(1-\theta) \Longrightarrow$ total number of agents is Nn^t.

Productivity

- Individual Productivity: Declining lifecycle pattern $\Phi_a = \kappa\chi^a$.
- Aggregate Productivity: Constant positive growth $g = T_t/T_{t-1}$ (must be equal across countries to avoid degenerate solutions).

Objective Function

$$\sum_{s=t}^{\infty} (\beta_t \theta)^{s-t} \left[\frac{1}{1-\gamma} \left(\left(c_{a+s,t+s}^{OLG} \right)^{\eta^{OLG}} \left(1 - \ell_{a+s,t+s}^{OLG} \right)^{1-\eta^{OLG}} \right)^{1-\gamma} \right]$$

Money: Assume cashless limit (Woodford, 2003).

Habit persistence generates aggregation problems.

A tractable version available in GIMF implies only weak consumption inertia.

That is why we also use retail quantity adjustment costs.

Consumption: $$c_{a,t}^{OLG} = \left(\int_0^1 \left(c_{a,t}^{OLG}(i) \right)^{\frac{\sigma_R - 1}{\sigma_R}} di \right)^{\frac{\sigma_R}{\sigma_R - 1}}$$

Financial Assets

1. Government bonds $B_{a,t}$ and domestic corporate bonds $B^N_{a,t}$, $B^T_{a,t}$
   (a) Complete home bias.
   (b) Denominated in domestic currency
2. Private bonds $F_{a,t}$
   (a) Only internationally traded asset.
   (b) Denominated in one currency (center country).
3. Insurance market for $B_{a,t}$ and $F_{a,t}$: $\Longrightarrow$ no myopia here.

Other Income Sources

1. Ownership of domestic firms and unions in twelve sectors N, T, D, C, I, R, U, M, K^N, K^T, EP^N, EP^T:

   (a) Complete home bias.

   (b) No traded equity, instead lump-sum dividend distributions $\longrightarrow$ firms are also myopic.

2. Labor:

   (a) Labor is sold to unions.

   (b) Declining lifecycle productivity: $\Phi_a = \kappa\chi^a$

Budget Constraint

$$P_t^R c_{a,t}^{OLG} + P_t^C c_{a,t}^{OLG} \tau_{c,t} + P_t \tau_{a,t}^{ls} + P_t \tau_{T_{a,t}}^{OLG} + B_{a,t} + B_{a,t}^N + B_{a,t}^T + \mathcal{E}_t F_{a,t}$$

$$= \frac{1}{\theta} \left[\frac{i_{t-1}}{(1+\xi_{t-1}^b)} \left(B_{a-1,t-1} + B_{a-1,t-1}^T + B_{a-1,t-1}^N \right) + i_{t-1}^* \mathcal{E}_t F_{a-1,t-1} (1+\xi_{t-1}^f) \right]$$

$$+ W_t \Phi_{a,t} \ell_{a,t}^{OLG} (1-\tau_{L,t}) + \sum_{j=N,T,D,C,I,R,U,M,X,F,K,EP} \int_0^1 D_{a,t}^j(i) di + P_t \Upsilon_{a,t}$$

Aggregation and Normalization

- Aggregation:

$$c_t^{OLG} = Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a c_{a,t}^{OLG}$$

- Normalization:

Normalization by Technology T_t and Population Growth Factor n^t:

$$\check{c}_t^{OLG} = c_t^{OLG}/T_t n^t$$

Note: We do NOT normalize by total population size Nn^t.

Therefore our variables are NOT in per capita terms.

FOC for Consumption-Leisure and UIP

$$\frac{\check{c}_t^{OLG}}{N(1-\psi) - \check{\ell}_t^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} \check{w}_t \frac{(1-\tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})}$$

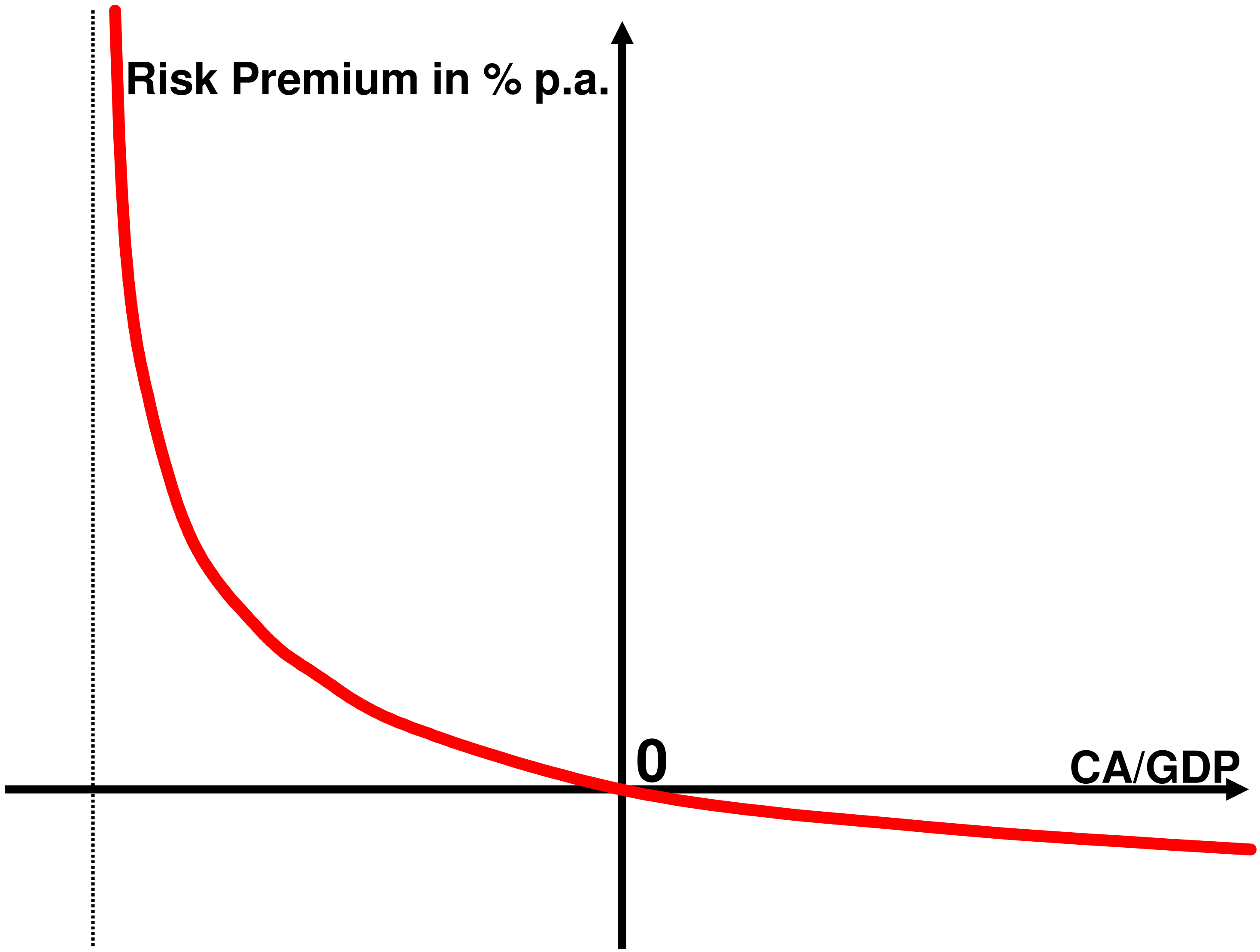
$$i_t = i_t^* \varepsilon_{t+1} (1+\xi_t^f)(1+\xi_t^b)$$

Foreign Exchange Risk Premium

$$\xi_t^f = y_1^{fx} + \frac{y_2^{fx}}{\left(cagdp_t^{filt} - y_4^{fx}\right)^{y_3^{fx}}} + S_t^{fx}$$

$$cagdp_t^{filt} = E_t \left(\Sigma_{k=k_l^{ca}}^{k_h^{ca}} 100 \frac{ca_{t+j}}{gdp_{t+j}} \right) / \left(k_h^{ca} - k_l^{ca} + 1\right)$$

- Depends in a nonlinear fashion on the current account to GDP ratio.
  - Almost flat at a positive CA ratio.
  - Extremely steep as the CA ratio approaches y_4^{fx} (a negative number).
- Zero risk premium at zero CA: $y_1^{fx} = -y_2^{fx} / \left(-y_4^{fx}\right)^{y_3^{fx}}$.
- In GIMF this function is NOT required for pinning down the steady state. It is there for realism.



Foreign Exchange Risk Premium

Aggregated Key FOC

$$\check{c}_t^{OLG}\Theta_t = \check{f}w_t + \check{h}w_t^L + \check{h}w_t^K$$

$$\text{Fin. Wealth} \quad : \quad \check{f}w_t = \frac{1}{\pi_t g n}\left[\frac{i_{t-1}}{\left(1+\xi_{t-1}^b\right)}\left(\check{b}_{t-1} + \check{b}_{t-1}^N + \check{b}_{t-1}^T\right)\right.$$
$$\left. + i_{t-1}^*(1+\xi_{t-1}^f)\varepsilon_t \check{f}_{t-1} e_{t-1}\right]$$

$$\text{Human Wealth 1: } \check{h}w_t^L = \left(N(1-\psi)(\check{w}_t(1-\tau_{L,t}))\right) + \tilde{E}_t \frac{\theta\chi g}{\check{r}_{t+1}}\check{h}w_{t+1}^L$$

$$\text{Human Wealth 2: } \check{h}w_t^K = \left(\sum_{j=Sector}\check{d}_t^j + Net\ Trf_t^{OLG}\right) + \tilde{E}_t \frac{\theta g}{\check{r}_{t+1}}\check{h}w_{t+1}^K$$

$$\text{Inverse of MPC:} \quad \Theta_t = \frac{p_t^R + p_t^C \tau_{c,t}}{\eta^{OLG}} + \tilde{E}_t \frac{\theta j_t}{\check{r}_{t+1}}\Theta_{t+1}$$

- Large discount factors ($\theta, \chi < 1$) = **highly non-Ricardian**.

Optimal Consumption - The Intuition

Example: Debt increase through initial tax cut.

- Lower taxes today, higher taxes tomorrow.

- Government: *Unchanged PDV of taxes at the market interest rate* r_t.

- Households: *Higher PDV of human wealth evaluated at subjective interest rate* r_t/θ or $r_t/\theta\chi$.
  - Short run effect: Higher consumption.
  - Long run effect: Lower consumption.

- Government debt that today's households do not expect to repay themselves (through taxes) is net wealth.

- More household myopia $\Longrightarrow$ government debt represents more net wealth.

The Marginal Propensity to Consume

Steady State Analysis

$$mpc^{ss} = \frac{\eta^{OLG}}{\bar{p}^R + \tau_c}\left(1 - \theta\beta^{\frac{1}{\gamma}} g^{(1-\eta^{OLG})(1-\frac{1}{\gamma})} \bar{r}^{\left(\frac{1}{\gamma}-1\right)}\right)$$

1. **Real interest rate and intertemporal EoS**: Empirically relevant case is intertemporal EoS<<1 ($\gamma >> 1$):

$$\frac{\partial mpc^{ss}}{\partial \bar{r}} > 0$$

   - With low intertemporal EoS, the income effect of higher r is stronger than the substitution effect.
   - Higher mpc can partly offset the reduction in $wealth$ due to higher r.

2. **Consumption taxes**: Higher τ_c reduces mpc, all other taxes reduce wealth.

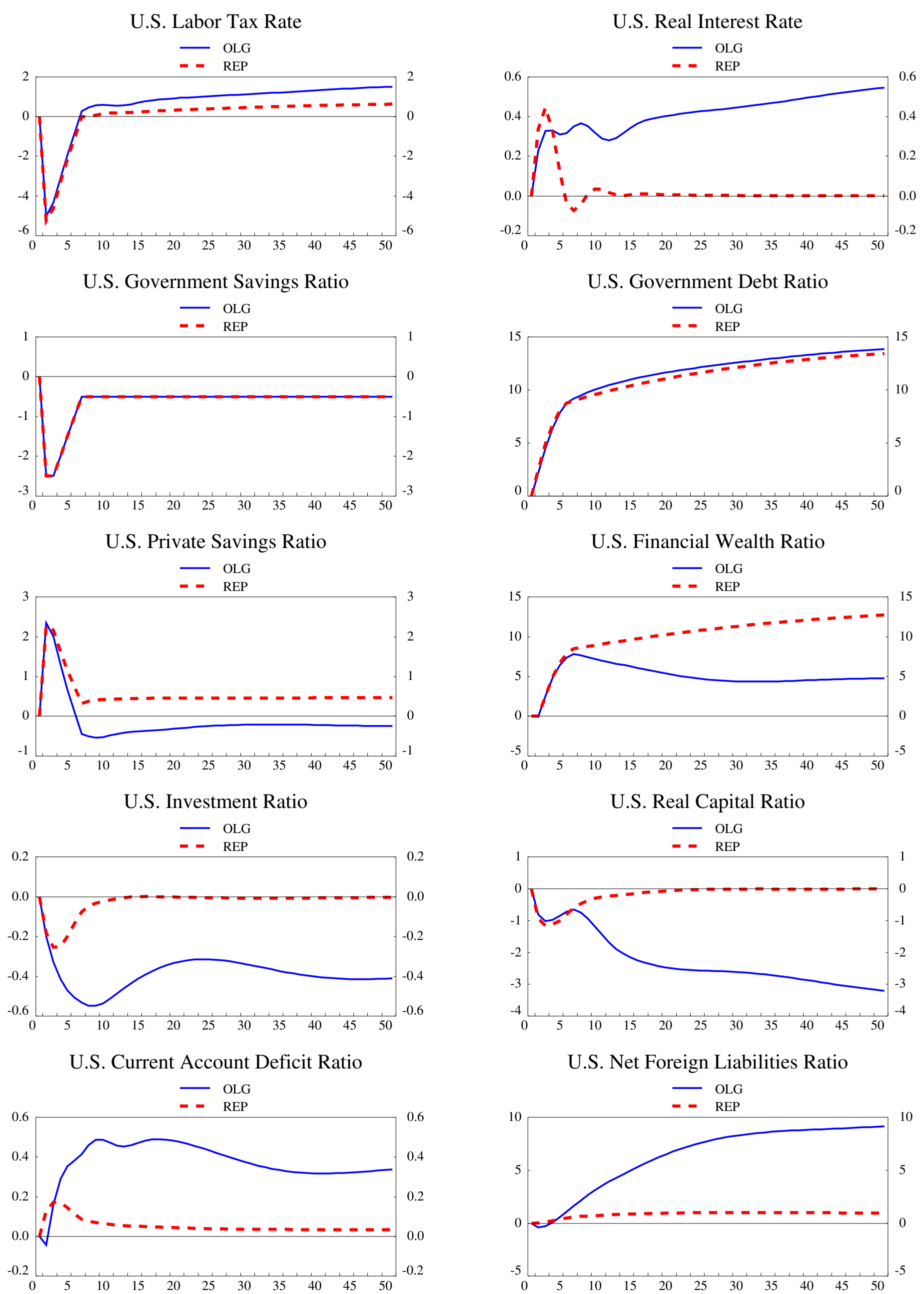
The Infinite Horizon Representative Agent Alternative

- Changed assumptions:
  1. OLG households replaced by infinitely lived households: $\theta = \chi = 1$.
  2. LIQ households have a higher population share to have sufficient "aggregate myopia".
  3. Steady state net foreign assets pinned down by the foreign exchange risk premium function.

- New consumption Euler equation:

$$\check{c}_{t+1}^{OLG} = \frac{j_t}{g} \check{c}_t^{OLG}$$

Illustration: GIMF and Fiscal Policy


U.S. Labor Tax Rate
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U.S. Real Capital Ratio
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U.S. Current Account Deficit Ratio
OLG
REP
U.S. Net Foreign Liabilities Ratio
OLG
REP

3.1 Liquidity Constrained Households

- Objective Function - 2 Possibilities:

  1. Same as OLG households: Intratemporal maximizers.

  2. Exogenous labor supply: Rule of thumb agents.

- Different constraint = not intertemporal = **highly non-Ricardian**:

$$\check{c}_t^{LIQ}(p_t^R + p_t^C \tau_{c,t}) = \check{w}_t \ell_t^{LIQ}(1 - \tau_{L,t}) + Net\ Trf_t^{LIQ}$$

- FOC for Intratemporal Maximizers:

$$\frac{\check{c}_t^{LIQ}}{N\psi - \check{\ell}_t^{LIQ}} = \frac{\eta^{LIQ}}{1 - \eta^{LIQ}} \check{w}_t \frac{(1 - \tau_{L,t})}{(p_t^R + p_t^C \tau_{c,t})}$$

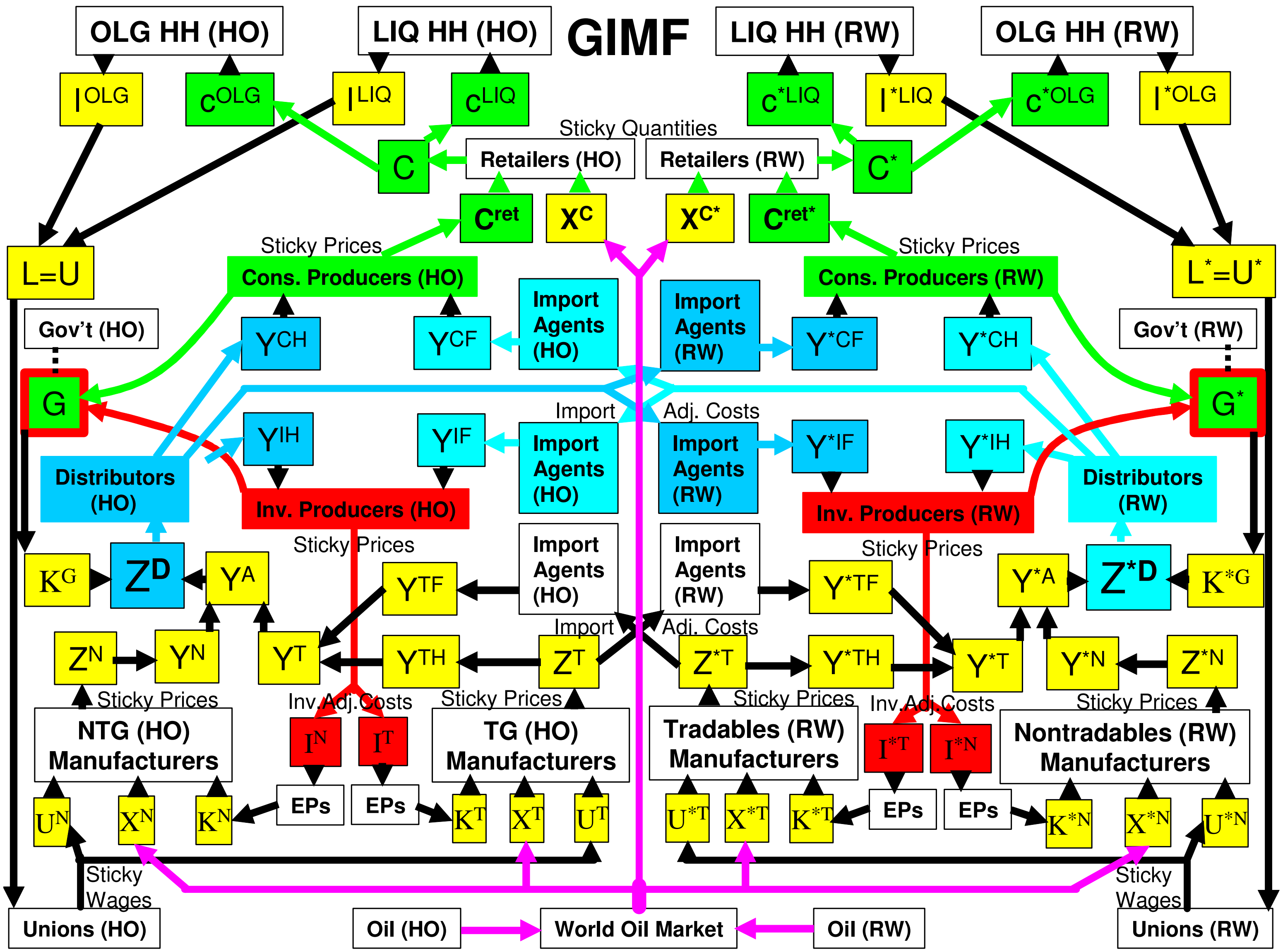
3.2 Aggregate Households

$$\check{L}_t = \check{\ell}_t^{OLG} + \check{\ell}_t^{LIQ}$$

$$\check{C}_t = \check{c}_t^{OLG} + \check{c}_t^{LIQ}$$

4 Firms and Unions

- **Competition:** Perfectly competitive in input markets, monopolistically competitive in output markets.
- **Rigidities:**
  - Nominal rigidities for manufacturers (N, T), unions (U), import agents (M), distributors (C, I).
  - Real rigidities for retailers (R).
  - No rigidities for input distributors (D), capital producers (K) and entrepreneurs (EP).
- **Fixed Cost of Production for N/T-Manufacturers and C/I-Distributors:** To calibrate aggregate steady state labor and capital shares.
- **Dividends (net cash flow):** Paid as lump-sum dividends to OLG households. That way firms can be modeled as myopic.
- **Discount Factor for PDV of Dividends:** $\theta/r_t = \beta\theta\left(\lambda_{a+1,t+1}/\lambda_{a,t}\right)$ (=pricing kernel of households).



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5 Manufacturers (N =Nontraded, T =Traded)

- **Output Demands**:
  1. Domestic Distributors.
  2. Import Agents abroad.

$$Z_t^N(i) = \left(\frac{P_t^N(i)}{P_t^N}\right)^{-\sigma_N} Z_t^N$$

- **Factor Demands:**
  1. Labor U_t^N from Unions.
  2. Utilized Capital K_{t-1}^N from Entrepreneurs.
  3. Oil X_t^N from World Oil Market.

- **Production Function:**

$$Z_t^N(i) = \mathfrak{T} * CES\left\{M_t^N(i),\ X_t^N(i)\left(1 - G_{X,t}^N(i)\right)\right\}$$

$$M_t^N(i) = CES\left\{K_{t-1}^N(i),\ T_t A_t^N U_t^N(i)\right\}$$

  - $\mathcal{T}$ = scale factor to calibrate relative per capita GDPs.
  - T_t = labor augmenting unit root technology to get balanced growth.
  - A_t^N = labor augmenting stationary technology.
  - $G_{X,t}^N$ = oil input adjustment cost.

Cash Flow Maximization

- **Objective Function:**

$$\underset{\{P_{t+s}^N(i), U_{t+s}^N(i), K_{t+s}^N(i)\}_{s=0}^{\infty}}{Max} \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} D_{t+s}^N(i)$$

- **Nominal Discount Factor (myopia):**

$$\tilde{R}_{t,s} = \Pi_{l=1}^{s} \frac{\theta}{i_{t+l-1}}$$

- **Dividends:**

$$D_t^N(i) = \left[P_t^N(i) Z_t^N(i) - V_t U_t^N(i) - P_t^X X_t^N(i) - R_{k,t}^N K_{t-1}^N(i) - Adj.Costs\right]$$

- **Rotemberg Inflation Adjustment Costs (identical in form in all sectors):**

$$G_{P,t}^N(i) = \frac{\phi_{P^N}}{2} Z_t^N \left(\frac{\frac{P_t^N(i)}{P_{t-1}^N(i)}}{\frac{P_{t-1}^N}{P_{t-2}^N}} - 1 \right)^2$$

- **Sticky Inflation FOC:** λ_t^N = marginal cost, p_t^N = output price, π_t^N = output price inflation.

$$\left[\frac{\sigma_N}{\sigma_N - 1} \frac{\lambda_t^N}{p_t^N} - 1 \right] = \frac{\phi_{P^N}}{\sigma_N - 1} \left(\frac{\pi_t^N}{\pi_{t-1}^N} \right) \left(\frac{\pi_t^N}{\pi_{t-1}^N} - 1 \right)$$
$$- \frac{\theta g n}{\check{r}_{t+1}} \frac{\phi_{P^N}}{\sigma_N - 1} \frac{p_{t+1}^N}{p_t^N} \frac{\check{Z}_{t+1}^N}{\check{Z}_t^N} \left(\frac{\pi_{t+1}^N}{\pi_t^N} \right) \left(\frac{\pi_{t+1}^N}{\pi_t^N} - 1 \right)$$

- **Labor Demand FOC:**

$$\check{v}_t = \lambda_t^N \check{F}_{U,t}^N$$

- **Oil Demand FOC:**

$$p_t^X = \lambda_t^N \check{F}_{X,t}^N$$

- **Capital Demand FOC:**

$$r_{k,t}^N = \lambda_t^N F_{K,t}^N$$

6 Capital Producers (N =Nontraded, T =Traded)

- **Intra-Period Optimizers:** Buy old depreciated capital from entrepreneurs $\Longrightarrow$ add investment $\Longrightarrow$ sell old+new capital back to entrepreneurs.

- **Capital Accumulation:** $\tilde{K}_t^N$ = intra-period physical capital (to be distinguished from utilized capital K_t^N and from time-dated physical capital $\bar{K}_t^N$)

$$\tilde{K}_t^N = \tilde{K}_{t-1}^N + S_t^{inv} I_t^N$$

- **Investment Adjustment Costs:**

$$G_{I,t}^N = \frac{\phi_I}{2} I_t^N \left(\frac{(I_t^N/(gn)) - I_{t-1}^N}{I_{t-1}^N} \right)^2$$

- **Optimization:**
  - Q_t^N = market value of existing capital (Tobin's q).
  - P_t^I = producer price of new investment.

$$\underset{\{I_{t+s}^N\}_{s=0}^{\infty}}{Max} E_t \boldsymbol{\Sigma}_{s=0}^{\infty} \tilde{R}_{t,s} \Big[Q_{t+s}^N \left(\tilde{K}_{t+s-1}^N + S_{t+s}^{inv} I_{t+s}^N \right) - Q_{t+s}^N \tilde{K}_{t+s-1}^N$$
$$- P_{t+s}^I \left(I_{t+s}^N + G_{I,t+s}^N \right) \Big]$$

- **Investment FOC:**

$$q_t^N S_t^{inv} = p_t^I + Investment\ Adj.\ Cost\ Terms_t$$

- p_t^I is sticky but q_t^N can jump $\Longrightarrow$ firm valuation effects.

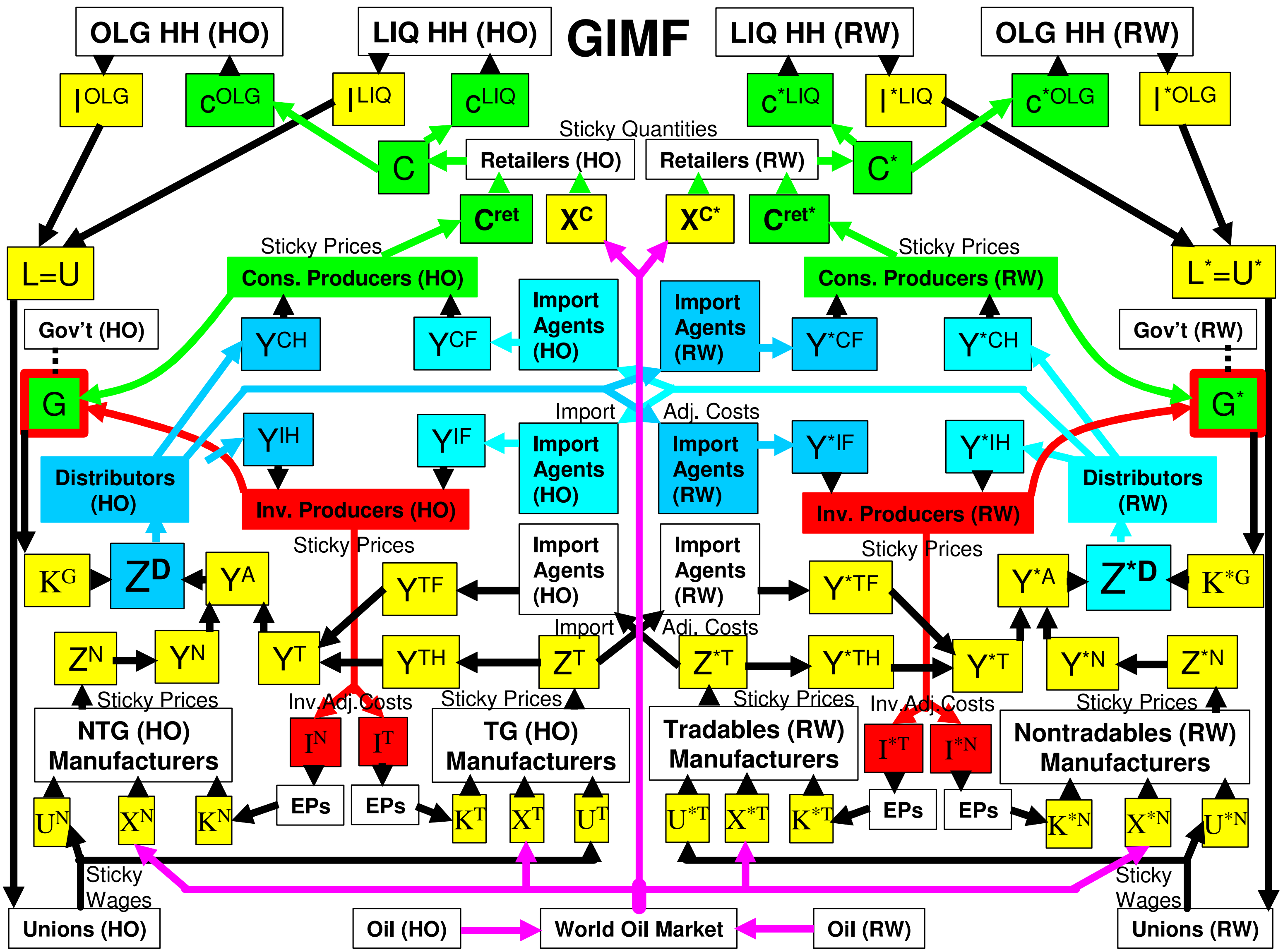
- Accumulation of Physical Capital:

$$\bar{K}_t^N = \left(1 - \delta_{K_t}^N\right) \bar{K}_{t-1}^N + S_t^{inv} I_t^N$$

- Time-varying Depreciation Rate:

  – S_t^{nwkshk} = capital destroying net worth shock

$$\delta_{K_t}^N = \bar{\delta}_K^N + S_t^{nwkshk}$$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Oil (HO)
World Oil Market
Oil (RW)
Unions (RW)

7 Entrepreneurs (N =Nontraded, T =Traded)

Capital Utilization Decision

- **Optimization:**
  - u_t^N = utilization rate of capital.
  - $a(u_t^N)$ = utilization cost function.
  - ω_t^N = idiosyncratic productivity shock.

$$\underset{u_t^N}{Max} \quad \left[u_t^N r_{k,t}^N - a(u_t^N)\right]\left(1 - \tau_{k,t}\right)\omega_t^N \bar{K}_{t-1}^N$$

- **FOC:**

$$r_{k,t}^N = a'(u_t^N) = \phi_a^N \sigma_a^N u_t^N + \phi_a^N\left(1 - \sigma_a^N\right)$$

Capital Utilization Decision continued

- **Utilization Cost Function $a(u_t^N)$:**

$$a(u_t^N) = \frac{1}{2}\phi_a^N \sigma_a^N \left(u_t^N\right)^2 + \phi_a^N \left(1 - \sigma_a^N\right) u_t^N + \phi_a^N \left(\frac{\sigma_a^N}{2} - 1\right)$$

- **Financial Return to Capital $ret_{k,t}^N$:**

$$ret_{k,t}^N = \frac{\left(u_t^N r_{k,t}^N - a(u_t^N) + \left(1 - \delta_{K_t}^N\right) q_t^N\right) - \tau_{k,t}\left(u_t^N r_{k,t}^N - a(u_t^N) - \delta_{K_t}^N q_t^N\right.}{q_{t-1}^N}$$

- **Relationship of Physical and Utilized Capital:**

$$K_t^N = u_t^N \bar{K}_t^N$$

Optimal Loan Contract

- Balance Sheet Identity (j = individual entrepreneur):
  - $B_t^N(j)$ = nominal debt.
  - $N_t^N(j)$ = nominal net worth.

$$B_t^N(j) = Q_t^N \bar{K}_t^N(j) - N_t^N(j)$$

- Default Cut-Off Productivity Level $\bar{\omega}_{t+1}^N$:
  - $\sigma_t^N = sd\left(\ln\left(\omega_t^N\right)\right)$.
  - $i_{B,t+1}^N$ = gross non-default loan rate.

$$\bar{\omega}_{t+1}^N ret_{k,t+1}^N Q_t^N \bar{K}_t^N(j) = i_{B,t+1}^N B_t^N(j)$$

- Lender's Zero-Profit Condition:
  - μ_t^N = bankruptcy cost as % of recoverable value

$$\begin{aligned}\check{i}_t B_t^N(j) &= \left(1 - F(\bar{\omega}_{t+1}^N)\right) i_{B,t+1}^N B_t^N(j) \\ &\quad + \left(1 - \mu_{t+1}^N\right) \int_0^{\bar{\omega}_{t+1}^N} Q_t^N \bar{K}_t^N(j) ret_{k,t+1}^N \omega f(\omega) d\omega\end{aligned}$$

- Lender's Gross Profits Share:

$$\Gamma(\bar{\omega}_{t+1}^N) = \int_0^{\bar{\omega}_{t+1}^N} \omega_{t+1}^N f(\omega_{t+1}^N) d\omega_{t+1}^N + \bar{\omega}_{t+1}^N \int_{\bar{\omega}_{t+1}^N}^{\infty} f(\omega_{t+1}^N) d\omega_{t+1}^N$$

- Lender's Monitoring Cost:

$$\mu_{t+1}^N G(\bar{\omega}_{t+1}^N) = \mu_{t+1}^N \int_0^{\bar{\omega}_{t+1}^N} \omega_{t+1}^N f(\omega_{t+1}^N) d\omega_{t+1}^N \ .$$

- Entrepreneur's Optimization Problem:

$$\underset{\bar{K}_t^N(j),\bar{\omega}_{t+1}^N}{Max} \left(1-\Gamma(\bar{\omega}_{t+1}^N)\right) ret_{k,t+1}^N Q_t^N \bar{K}_t^N(j)$$

$$+\lambda_t \left\{ \left(\Gamma(\bar{\omega}_{t+1}^N) - \mu_{t+1}^N G(\bar{\omega}_{t+1}^N)\right) ret_{k,t+1}^N Q_t^N \bar{K}_t^N(j)\right.$$

$$\left. -\check{i}_t \left(Q_t^N \bar{K}_t^N(j) - N_t^N(j)\right)\right\}$$

- Results: A set of **NONLINEAR** optimality conditions
  1. Zero profit condition for lenders.
  2. Upward-sloping supply curve of external funds B_t^N for entrepreneurs.
     - Shape depends on size of bankruptcy costs μ_t^N.
     - Shape depends on distribution of productivities σ_t^N.
  3. Negative shocks to net worth $S_t^{N,nwd}, S_t^{N,nwy}, S_t^{N,nwk}$:
     - Increase the external finance premium and take you into a steeper part of the curve.
     - Have very persistent effects because net worth takes time to be rebuilt.

- Aggregate Net Worth Evolution:
  - $d\check{i}v_t^N =$ Dividends to OLG Households.
  - $\check{S}_t^{N,nwy} =$ output destroying net worth shock

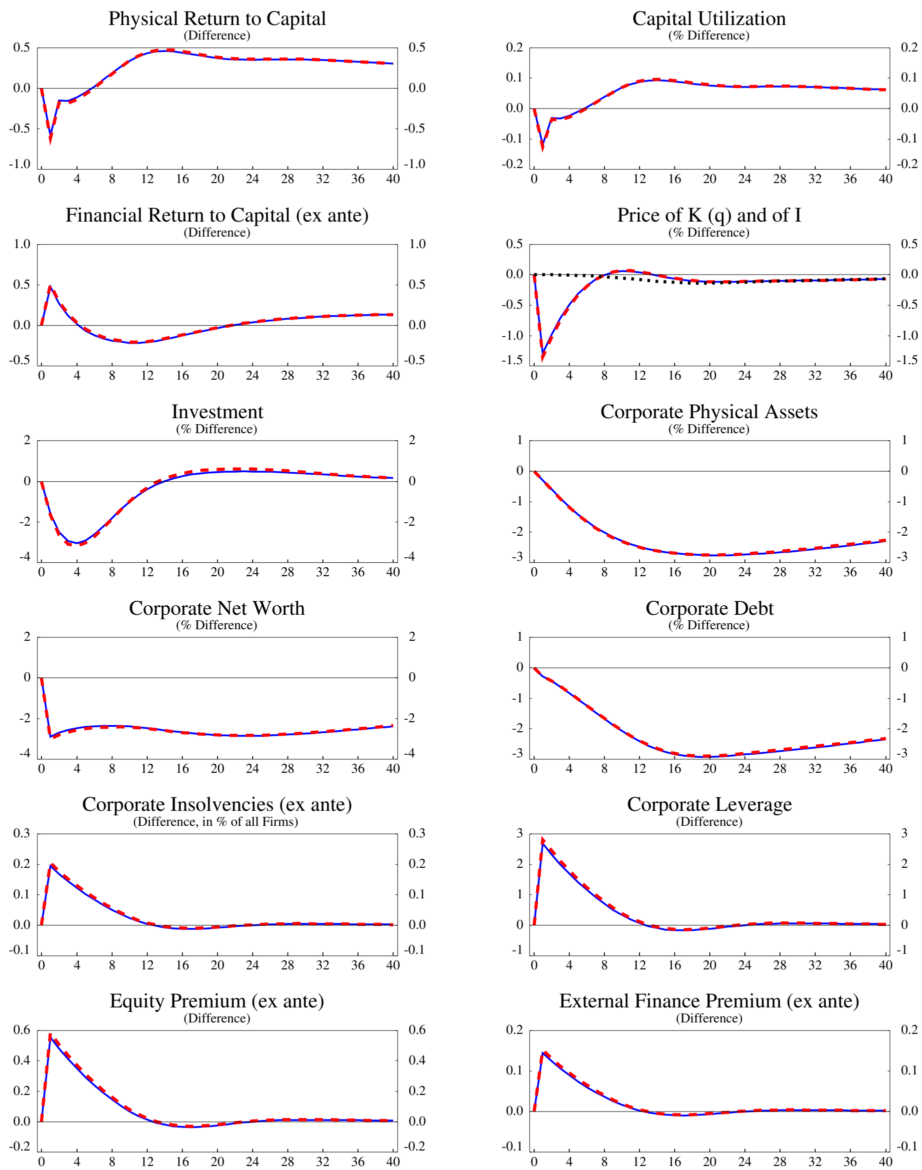
$$
\begin{aligned}
\check{n}_t^N &= \frac{\check{r}_t}{gn}\check{n}_{t-1}^N + q_{t-1}^N \bar{\check{K}}_{t-1}^N \left(\frac{r\check{e}t_{k,t}^N}{gn}\left(1-\mu_t^N G_t^N\right) - \frac{\check{r}_t}{gn} \right) \\
&\quad -p_t^N \left(d\check{i}v_t^N + \check{S}_t^{N,nwy} \right)
\end{aligned}
$$

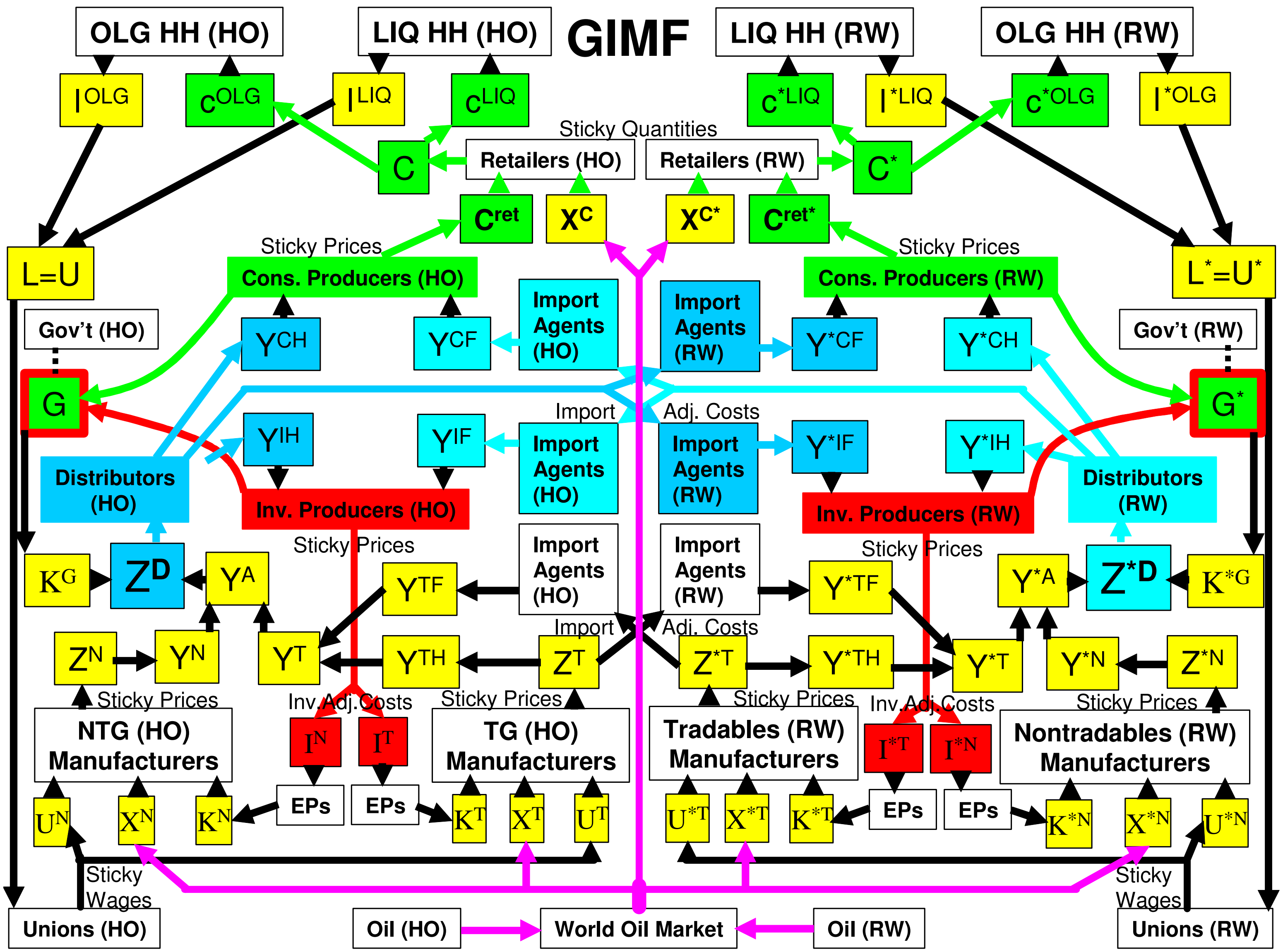
- Dividend Process: $$\check{div}_t^N = \check{inc}_t^N + \theta_{nw}^N \left(\check{n}_t^N - \check{n}_t^{N,filt}\right)$$
  - Regular Dividend: $S_t^{N,nwd} =$ dividend related net worth shock
$$p_t^N \ \check{inc}_t^N \ = \ E_t \frac{S_t^{N,nwd}}{\left(k_h^{incN} - k_l^{incN} + 1\right)} \Sigma_{k=k_l^{incN}}^{k_h^{incN}} \left[\check{n}_{t+j}^N + p_{t+j}^N \left(\check{div}_{t+j}^N + \check{S}_{t+j}^{N,nwyshk}\right)\right]$$
  - Net Worth Rebuilding if $\theta_{nw}^N > 0$:
$$\check{n}_t^{N,filt} = E_t \Sigma_{k=k_l^{nw}}^{k_h^{nw}} \left(\check{n}_{t+j}^N\right) / \left(k_h^{nw} - k_l^{nw} + 1\right)$$
- Aggregate Entrepreneur Dividends to Households:
$$\check{d}_t^{EP} = p_t^N \check{div}_t^N + p_t^{TH} \check{div}_t^T$$

Illustration: GIMF and Macro-Financial Linkages

Net Worth Shock C - Capital Destruction
Capital in Home

T __ and N - -





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Sticky Wages
Sticky Wages
Unions (HO)
Oil (HO)
World Oil Market
Oil (RW)
Unions (RW)

8 Oil Producers

- Homogenous good worldwide.

- Stochastic Endowment: $\check{X}_t^{sup}$.

- Manufacturers' and Households' Demand: $\check{X}_t^{dem} = \check{X}_t^T + \check{X}_t^N + \check{X}_t^C$.

- Price p_t^X, determined in perfectly competitive world market.

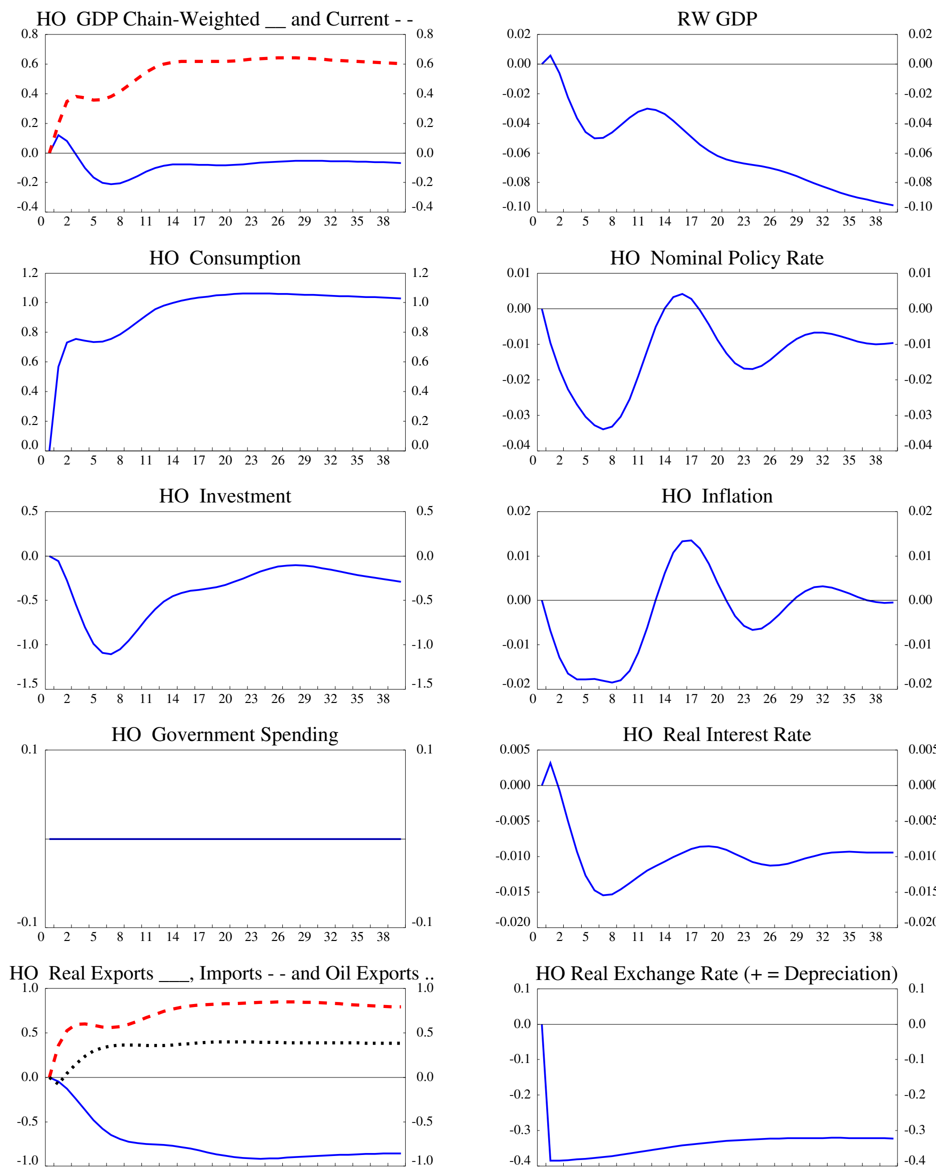
- Worldwide Market Clearing:

$$\Sigma_{j=1}^{\tilde{N}} \left(\check{X}_t^{sup(j)} - \check{X}_t^{dem(j)} \right) = 0$$

- Revenue Shares:
  1. Fixed share of steady state revenue to domestic factors:
  $$\bar{d}^X = s_d^x \bar{p}^X \bar{X}^{sup}$$
  2. Fixed share of remainder to foreigners:
  $$\check{f}_t^X(1,j) = s_f^x(1,j)\left(p_t^X \check{X}_t^{sup} - \bar{d}^X\right)$$
  $$\check{f}_t^X = \check{f}_t^X(1) = \Sigma_{j=2}^{\tilde{N}} \check{f}_t^X(1,j)$$
  3. Remainder to domestic government:
  $$\check{g}_t^X = p_t^X \check{X}_t^{sup} - \bar{d}^X - \check{f}_t^X$$

Illustration: GIMF and Oil

Oil Demand Shock
HO Survey



9 Unions

- **Output Demands:** Labor demands by manufacturers.

$$U_t(i) = \left(\frac{V_t(i)}{V_t}\right)^{-\sigma_U} U_t$$

- **Factor Demands:** Labor supply from households.

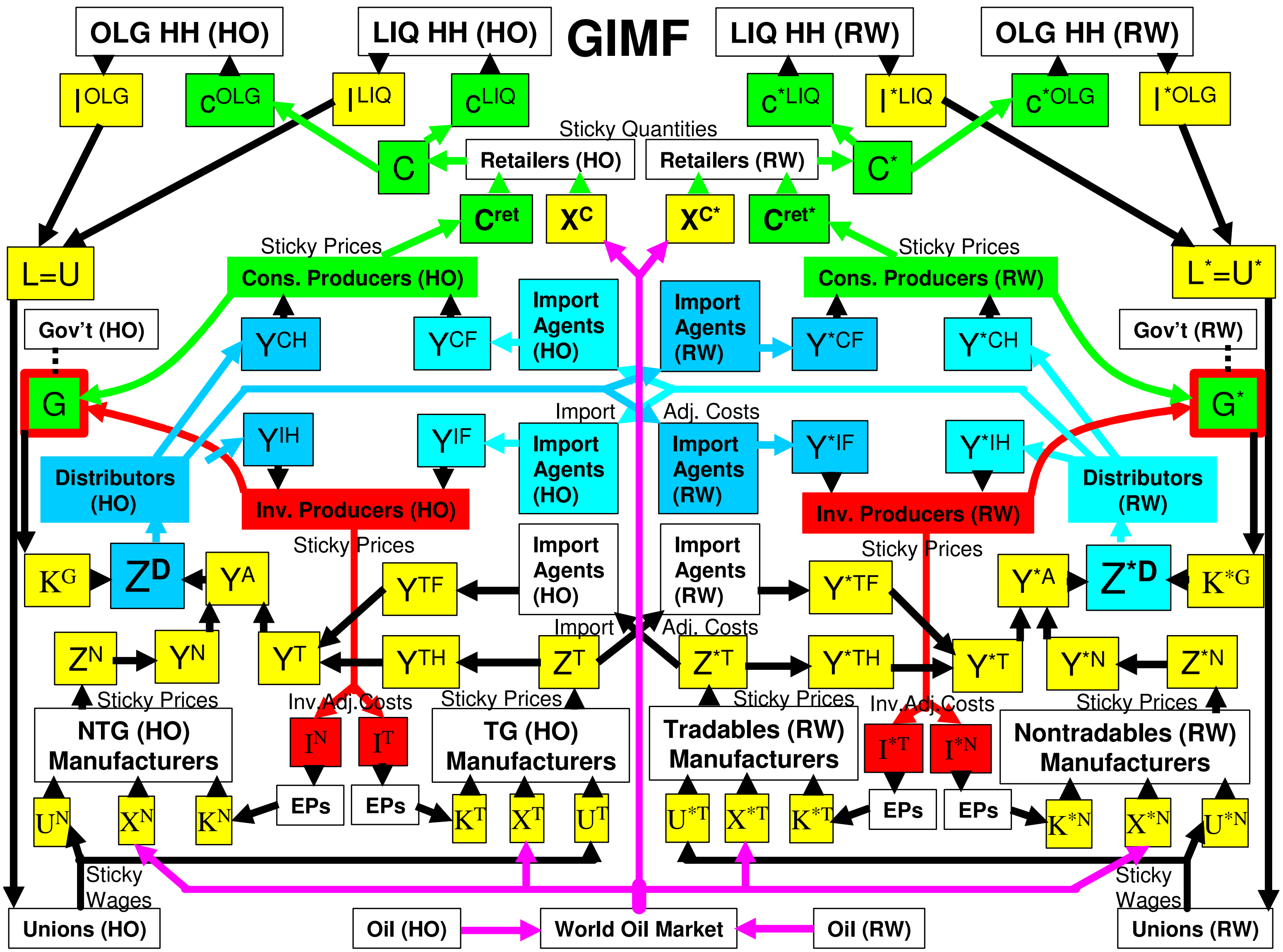
- **Sticky Nominal (Producer) Wages**: Locating sticky wages in unions rather than households is critical for aggregation.

- **Optimization (subject to nominal rigidities):**

$$\underset{\{V_{t+s}(i)\}_{s=0}^{\infty}}{Max} \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(V_{t+s}(i) - W_{t+s}\right) U_{t+s}(i) - V_{t+s} G_{P,t+s}^{U}(i) \right]$$

- **Sticky Wage Inflation:**

$$\left[\mu_t^U \frac{\check{w}_t}{\check{v}_t} - 1\right] = \phi_{P^U} \left(\mu_t^U - 1\right) \left(\frac{\pi_t^V}{\pi_{t-1}^V}\right) \left(\frac{\pi_t^V}{\pi_{t-1}^V} - 1\right)$$
$$- E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_{P^U} \left(\mu_t^U - 1\right) \frac{\check{v}_{t+1}}{\check{v}_t} \frac{\check{U}_{t+1}}{\check{U}_t} \left(\frac{\pi_{t+1}^V}{\pi_t^V}\right) \left(\frac{\pi_{t+1}^V}{\pi_t^V} - 1\right)$$



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Unions (RW)

10 Import Agents

- An individual import agent only imports the goods of one country.
- This import agent is owned by that country.
- Import agent sells to distributors subject to nominal rigidities: Pricing-to-market or local currency pricing.
- Three continua of import agents, distinguished by type of import:
  1. Intermediate tradables T (used for illustration below).
  2. Final consumption goods C.
  3. Final investment goods I.

- **Output Demands:** Goods demands by distributors.

$$Y_t^{TM}(i) = \left(\frac{P_t^{TM}(i)}{P_t^{TM}}\right)^{-\sigma_{TM}} Y_t^{TM}$$

- **Factor Demands (from each source country):** Intermediate goods Z_t^T, final goods Z_t^C, Z_t^I.

- Marginal cost of import agent = CIF price of imported good:

$$p_t^{TM,cif} = p_t^{TH^*} e_t$$

- **Optimization (subject to nominal rigidities = local currency pricing):**

$$\underset{\left\{P_{t+s}^{TM}(i)\right\}_{s=0}^{\infty}}{Max} \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}^{TM}(i) - P_{t+s}^{TM,cif} \right) Y_{t+s}^{TM}(i) - P_{t+s}^{TM} G_{P,t+s}^{TM}(i) \right]$$

- **Sticky Import Price Inflation:**

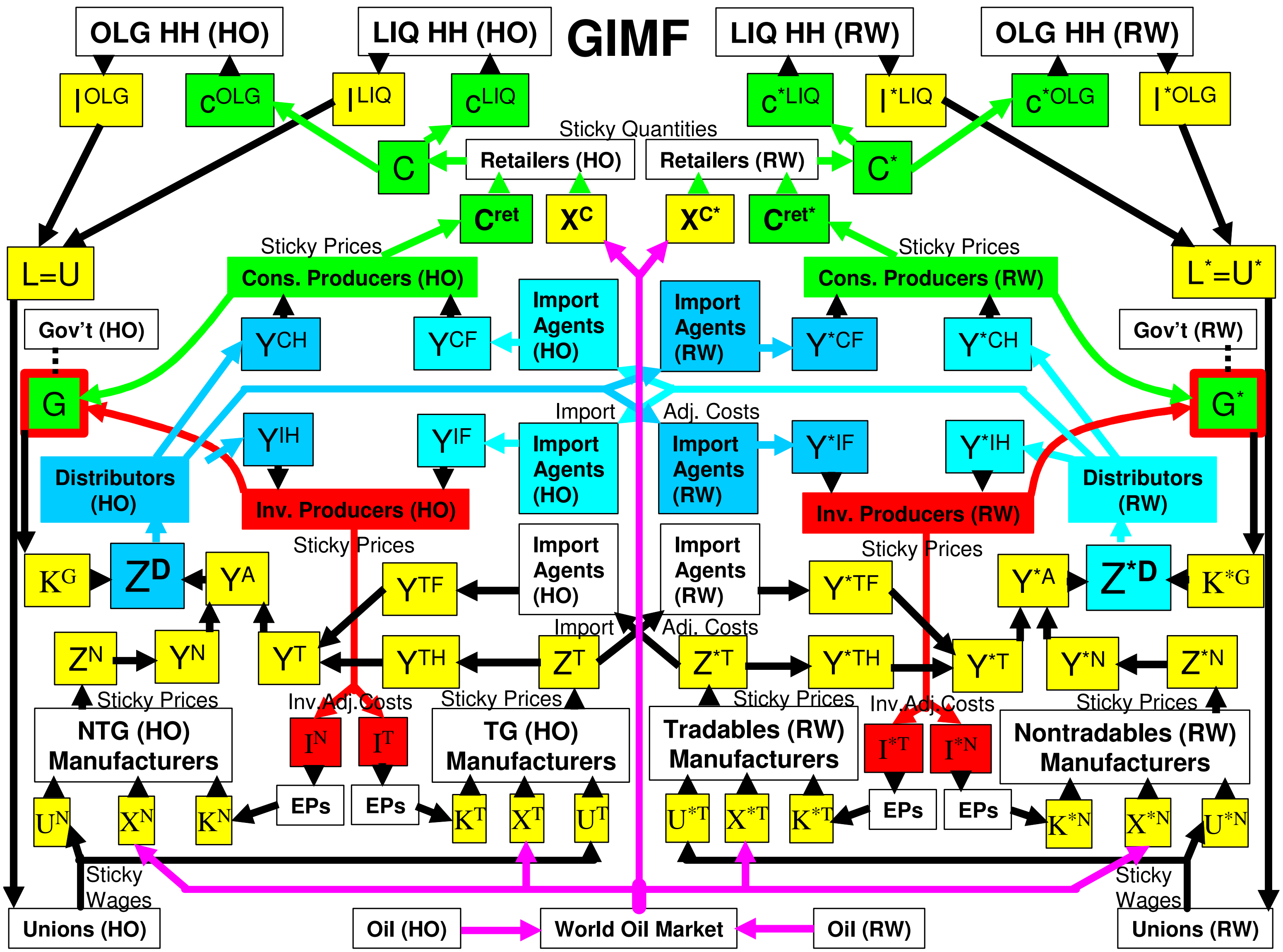
$$\left[\frac{\sigma_{TM}}{\sigma_{TM}-1} \frac{p_t^{TM,cif}}{p_t^{TM}} - 1 \right] = \frac{\phi_{P^{TM}}}{\sigma_{TM}-1} \left(\frac{\pi_t^{TM}}{\pi_{t-1}^{TM}} \right) \left(\frac{\pi_t^{TM}}{\pi_{t-1}^{TM}} - 1 \right)$$
$$-\frac{\theta g n}{\check{r}_{t+1}} \frac{p_{t+1}^{TM}}{p_t^{TM}} \frac{\check{Y}_{t+1}^{TM}}{\check{Y}_t^{TM}} \frac{\phi_{P^{TM}}}{\sigma_{TM}-1} \left(\frac{\pi_{t+1}^{TM}}{\pi_t^{TM}} \right) \left(\frac{\pi_{t+1}^{TM}}{\pi_t^{TM}} - 1 \right)$$

- Dividends paid out to OLG agents in the **EXPORT** country:

$$\check{d}_t^{TM}(HO) = \check{d}_t^{TM}(RW, HO) * e_t$$

  This becomes a sum over multiple dividends in the multi-country case.

- Implications:
  1. Producer country absorbs the exchange rate risk through import agent profits.
  2. Relative domestic prices change slowly even if the exchange rate jumps.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Unions (RW)

11 Input Distributors

Output Demands and Factors of Production

- **Output Demands:** (i) Consumption Goods Distributors. (ii) Investment Goods Distributors. (iii) Final Goods Import Agents.

- **Factor Demands:** (i) Domestic Manufacturers (N, T). (ii) Intermediates import agents. (iii) Government: Public capital stock (for free).

Production 1: Foreign Tradables Composite

Relevant only for versions of GIMF with three or more countries.

CES production function combining imports from all foreign countries into one foreign good.

Using an international trade matrix to calibrate shares.

Production 2: Home and Foreign Tradables Composite

$$Y_t^T(i) = \left(\left(\alpha_{H_t}^T\right)^{\frac{1}{\xi_T}} \left(Y_t^{TH}(i)\right)^{\frac{\xi_T - 1}{\xi_T}} \right.$$

$$\left. + \left(1 - \alpha_{H_t}^T\right)^{\frac{1}{\xi_T}} \left(Y_t^{TF}(i)(1 - G_{F,t}^T(i))\right)^{\frac{\xi_T - 1}{\xi_T}} \right)^{\frac{\xi_T}{\xi_T - 1}}$$

$$G_{F,t}^T(i) = \frac{\phi_{FT}}{2} \frac{\left(\mathcal{R}_t^T - 1\right)^2}{1 + \left(\mathcal{R}_t^T - 1\right)^2} \quad , \quad \mathcal{R}_t^T = \frac{\frac{Y_t^{TF}(i)}{Y_t^T(i)}}{\frac{Y_{t-1}^{TF}}{Y_{t-1}^T}}$$

Production 3: Tradables-Nontradables Composite

$$Y_t^A(i) = \left((1-\alpha_N)^{\frac{1}{\xi_A}} \left(Y_t^T(i) \right)^{\frac{\xi_A - 1}{\xi_A}} + (\alpha_N)^{\frac{1}{\xi_A}} \left(Y_t^N(i) \right)^{\frac{\xi_A - 1}{\xi_A}} \right)^{\frac{\xi_A}{\xi_A - 1}}$$

Production 4: Private Sector - Public Sector Composite

$$Z_t^D(i) = Y_t^A(i) \left(K_t^{G1}\right)^{\alpha_{G1}} \left(K_t^{G2}\right)^{\alpha_{G2}} \mathcal{S}$$

$$p_t^{DH} \left(K_t^{G1}\right)^{\alpha_{G1}} \left(K_t^{G2}\right)^{\alpha_{G2}} \mathcal{S} = p_t^A$$

- Constant returns to scale in private sector inputs.
- $K_t^{G1/2}$ enter like technology, with $\alpha_{G1/2}$ determining the elasticity of output with respect to $K_t^{G1/2}$.
- $\mathcal{S}$ is a scale factor that is used to fix steady state output at $\bar{Z}^D = \bar{Y}^A$.

12 Investment Goods Distributors

Output Demands and Factors of Production

- **Output Demands:** (i) Capital Goods Producers. (ii) Government. (iii) Adjustment Costs.

$$\mathcal{D}_t^I(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\sigma_D} \mathcal{D}_t^I$$

- **Factor Demands:** (i) Home Input Distributors. (ii) Foreign Input Distributors, via Import Agents..

Production: Home and Foreign Final Output Composite

$$Z_t^I(i) = \left((\alpha_{IH})^{\frac{1}{\xi_I}} \left(Y_t^{IH}(i) \right)^{\frac{\xi_I - 1}{\xi_I}} + (1 - \alpha_{IH})^{\frac{1}{\xi_I}} \left(Y_t^{IF}(i)(1 - G_{F,t}^I(i)) \right)^{\frac{\xi_I - 1}{\xi_I}} \right)^{\frac{\xi_I}{\xi_I - 1}}$$

$$G_{F,t}^I(i) = \frac{\phi_{FI}}{2} \frac{\left(\mathcal{R}_t^I - 1 \right)^2}{1 + \left(\mathcal{R}_t^I - 1 \right)^2} \quad , \quad \mathcal{R}_t^I = \frac{\frac{Y_t^{IF}(i)}{Z_t^I(i)}}{\frac{Y_{t-1}^{IF}}{Z_{t-1}^I}}$$

Profit Maximization

- **Optimization (subject to nominal rigidities):**

$$\underset{\{P_{t+s}(i)\}_{s=0}^{\infty}}{Max} E_t \boldsymbol{\Sigma}_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}^{ZI}(i) - P_{t+s}^{II} \right) \mathcal{D}_{t+s}^{I}(i) \right.$$
$$\left. - P_{t+s}^{ZI} G_{P,t+s}^{I}(i) - P_{t+s}^{ZI} T_{t+s} \omega^{I} \right]$$

- **FOC:**
  1. Sticky final investment goods inflation.
  2. Factor demands for Home and Foreign inputs.

- **Implications:**
  1. Cascading sticky inflation.
  2. Sluggish adjustment of imports.

Unit Root Shocks to Investment Goods Price

- Relative price $\tilde{p}_t^I$ = inverse of technology:

$$\tilde{p}_t^I = \frac{1}{T_t^I}$$

- Optimality and Market Clearing:

$$p_t^I = p_t^{ZI} \tilde{p}_t^I$$

$$Z_t^I = \tilde{p}_t^I \left(I_t^N + I_t^T + Y_t^{GI} + Adj.Costs \right)$$

13 Consumption Goods Distributors

Output Demands and Factors of Production

- **Output Demands:** (i) Consumption Goods Retailers. (ii) Government. (iii) Adjustment Costs.

$$\mathcal{D}_t^C(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\sigma_D} \mathcal{D}_t^C$$

- **Factor Demands:** (i) Home Distributors of Inputs. (ii) Foreign Distributors of Inputs.

Production: Home and Foreign Final Output Composite

$$Z_t^C(i) = \left((\alpha_{CH})^{\frac{1}{\xi_C}} \left(Y_t^{CH}(i) \right)^{\frac{\xi_C - 1}{\xi_C}} + (1 - \alpha_{CH})^{\frac{1}{\xi_C}} \left(Y_t^{CF}(i)(1 - G_{F,t}^C(i)) \right)^{\frac{\xi_C - 1}{\xi_C}} \right)$$

$$G_{F,t}^C(i) = \frac{\phi_{FC}}{2} \frac{\left(\mathcal{R}_t^C - 1 \right)^2}{1 + \left(\mathcal{R}_t^C - 1 \right)^2} \quad , \quad \mathcal{R}_t^C = \frac{\frac{Y_t^{CF}(i)}{Z_t^C(i)}}{\frac{Y_{t-1}^{CF}}{Z_{t-1}^C}}$$

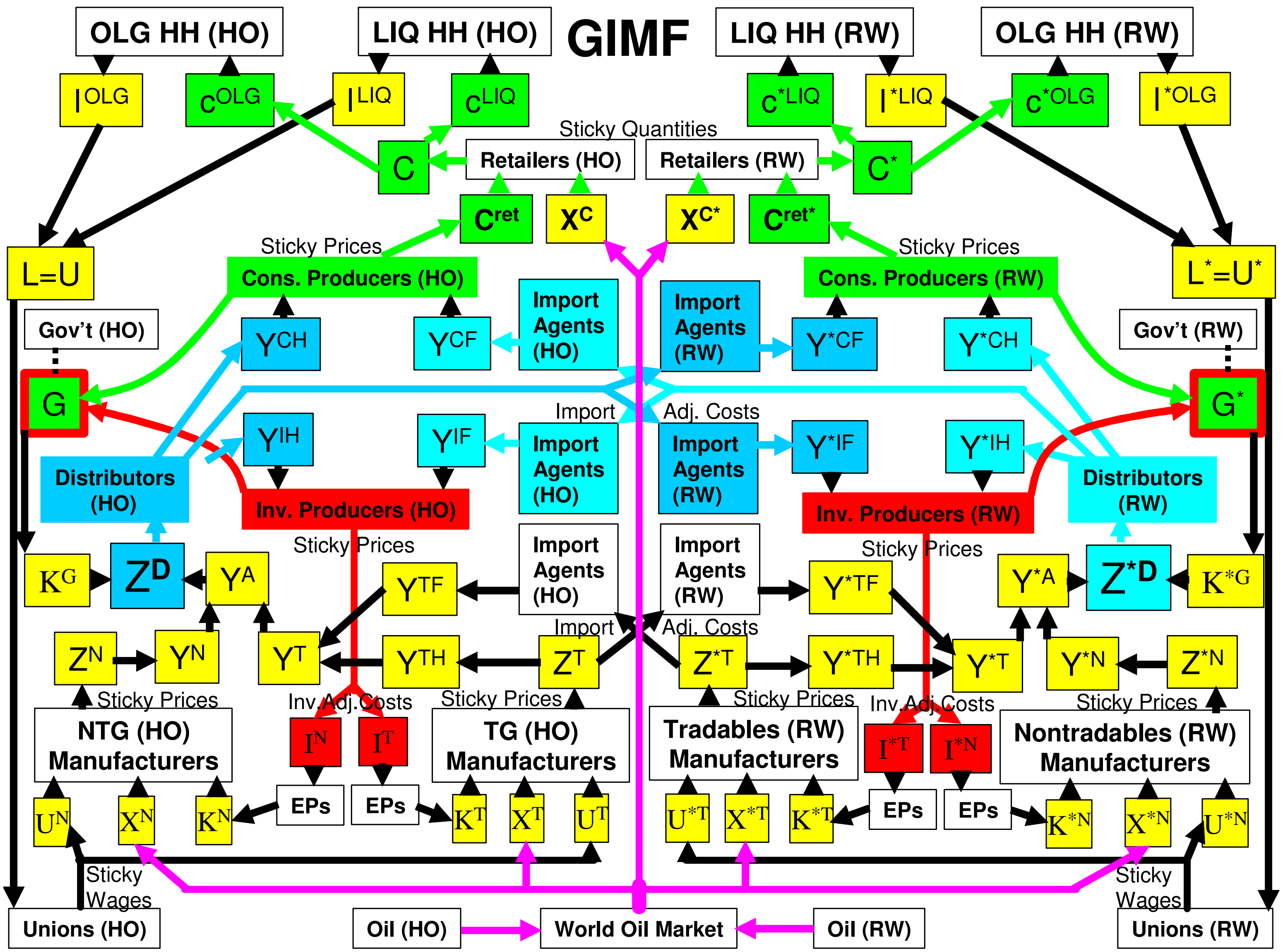
Profit Maximization

- **Optimization (subject to nominal rigidities):**

$$\underset{\{P_{t+s}(i)\}_{s=0}^{\infty}}{Max} E_t \boldsymbol{\Sigma}_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}(i) - P_{t+s}^{CC} \right) \mathcal{D}_{t+s}(i) \right.$$
$$\left. - P_{t+s} G_{P,t+s}^{C}(i) - P_{t+s} T_{t+s} \omega^{C} \right]$$

- **FOC:**
  1. Sticky final consumption goods inflation (this is the numeraire).
  2. Factor demands for Home and Foreign inputs.

- **Implications:**
  1. Cascading sticky inflation.
  2. Sluggish adjustment of imports.



GIMF
OLG HH (HO)
LIQ HH (HO)
LIQ HH (RW)
OLG HH (RW)
I^{OLG}
C^{OLG}
I^{LIQ}
C^{LIQ}
C^{*LIQ}
I^{*LIQ}
C^{*OLG}
I^{*OLG}
Sticky Quantities
C
Retailers (HO)
Retailers (RW)
C^{*}
C^{ret}
X^{C}
X^{C*}
C^{ret*}
Sticky Prices
Sticky Prices
$L=U$
Cons. Producers (HO)
Cons. Producers (RW)
$L^{*}=U^{*}$
Gov't (HO)
Y^{CH}
Y^{CF}
Import Agents (HO)
Import Agents (RW)
Y^{*CF}
Y^{*CH}
Gov't (RW)
G
G^{*}
Import
Adj. Costs
Y^{IH}
Y^{IF}
Import Agents (HO)
Import Agents (RW)
Y^{*IF}
Y^{*IH}
Distributors (HO)
Inv. Producers (HO)
Inv. Producers (RW)
Distributors (RW)
Sticky Prices
Import Agents (HO)
Import Agents (RW)
Sticky Prices
K^{G}
Z^{D}
Y^{A}
Y^{TF}
Y^{*TF}
Y^{*A}
Z^{*D}
K^{*G}
Import
Adj. Costs
Z^{N}
Y^{N}
Y^{T}
Y^{TH}
Z^{T}
Z^{*T}
Y^{*TH}
Y^{*T}
Y^{*N}
Z^{*N}
Sticky Prices
Inv. Adj. Costs
Sticky Prices
Sticky Prices
Inv. Adj. Costs
Sticky Prices
NTG (HO) Manufacturers
I^{N}
I^{T}
TG (HO) Manufacturers
Tradables (RW) Manufacturers
I^{*T}
I^{*N}
Nontradables (RW) Manufacturers
EPs
EPs
EPs
EPs
U^{N}
X^{N}
K^{N}
K^{T}
X^{T}
U^{T}
U^{*T}
X^{*T}
K^{*T}
K^{*N}
X^{*N}
U^{*N}
Sticky Wages
Sticky Wages
Unions (HO)
Oil (HO)
World Oil Market
Oil (RW)
Unions (RW)

14 Retailers

- **Production Function:** C_t (relative price p_t^C) combines final output with oil

$$
\begin{aligned}
C_t(i) \;=\; & \Bigg(\left(1-\alpha_{C_t}^X\right)^{\frac{1}{\xi_{XC}}} \left(C_t^{ret}(i)\right)^{\frac{\xi_{XC}-1}{\xi_{XC}}} \\
& + \left(\alpha_{C_t}^X\right)^{\frac{1}{\xi_{XC}}} \left(X_t^C(i)\left(1-G_{X,t}^C(i)\right)\right)^{\frac{\xi_{XC}-1}{\xi_{XC}}} \Bigg)^{\frac{\xi_{XC}}{\xi_{XC}-1}}
\end{aligned}
$$

- **Results:**
  - Factor Demands.
  - Sluggish adjustment of oil inputs due to adjustment cost $G_{X,t}^C$.

- **Output Demands:** Final goods demands by households.

$$C_t(i) = \left(\frac{P_t^R(i)}{P_t^R} \right)^{-\sigma_R} C_t$$

- **Factor Demands:** (i) Final goods from consumption goods distributors. (ii) Oil from World Oil Market.

- **Sales Adjustment Costs:**

$$G_{C,t}(i) = \frac{\phi_C}{2} C_t \left(\frac{(C_t(i)/(gn)) - C_{t-1}(i)}{C_{t-1}(i)} \right)^2$$

  - These help to generate sluggishness in consumption.
  - Habit persistence with our household preferences is not very powerful, but we need those preferences for easy aggregation.

- **Optimization:**

$$\underset{\left\{P^R_{t+s}(i)\right\}^{\infty}_{s=0}}{Max} \Sigma^{\infty}_{s=0} \tilde{R}_{t,s} \left[P^R_{t+s}(i) C_{t+s}(i) - P^C_{t+s} C_{t+s}(i) - P_{t+s} G_{C,t+s}(i) \right]$$

- **Sticky Sales Volume:**

$$\left[\frac{\sigma_R - 1}{\sigma_R} \frac{p^R_t}{p^C_t} - 1 \right] = \phi_C \left(\frac{\check{C}_t - \check{C}_{t-1}}{\check{C}_{t-1}} \right) \frac{\check{C}_t}{\check{C}_{t-1}}$$

$$-E_t \frac{\theta g n}{\check{r}_{t+1}} \phi_C \left(\frac{\check{C}_{t+1} - \check{C}_t}{\check{C}_t} \right) \left(\frac{\check{C}_{t+1}}{\check{C}_t} \right)^2$$

15 Government

15.1 Fiscal Policy

Government Production

- Inputs: Consumption and investment goods:

$$Z_t^G = \left((\alpha_{GC})^{\frac{1}{\xi_G}} \left(Y_t^{GC}\right)^{\frac{\xi_G - 1}{\xi_G}} + (1 - \alpha_{GC})^{\frac{1}{\xi_G}} \left(Y_t^{GI}\right)^{\frac{\xi_G - 1}{\xi_G}} \right)^{\frac{\xi_G}{\xi_G - 1}}$$

- Unit root in relative price of government output:

$$p_t^G = p_t^{ZG} \tilde{p}_t^G$$

$$\check{G}_t^{cons} + \check{G}_t^{inv} = \check{G}_t = \left(\check{Z}_t^G / \tilde{p}_t^G \right)$$

Government Investment Spending

$$\check{K}_t^{G1} gn = (1 - \delta_G)\, \check{K}_{t-1}^{G1} + \check{G}_{t-1}^{inv}$$

Dividend Redistribution to LIQ Agents

$$\begin{aligned}\check{\tau}_{T,t} &= \iota \left(\check{d}_t^N + \check{d}_t^T + \check{d}_t^D + \check{d}_t^C + \check{d}_t^I + \check{d}_t^M + \bar{d}_t^X + \check{d}_t^F + \check{d}_t^K + \check{d}_t^{EP} \right) \\ & \quad + \frac{c_t^{LIQ}}{C_t} \left(\check{d}_t^R + \check{\Upsilon}_t - \check{\tau}_t^{ls} \right) + \frac{\ell_t^{LIQ}}{L_t} \check{d}_t^U\end{aligned}$$

$\iota \leq \psi$: LIQ agents get a smaller dividend share than their share in the population.

Government Budget Constraint

Evolution of Bonds: $\check{b}_t = \frac{i_{t-1}}{\pi_t g n} \check{b}_{t-1} - \check{s}_t$

Primary Surplus: $\check{s}_t = \check{\tau}_t + \check{g}_t^X - p_t^G \check{G}_t - \check{\Upsilon}_t$

Tax Revenue :

$$\check{\tau}_t = \tau_{L,t} \check{w}_t L_t + \tau_{c,t} p_t^C \check{C}_t$$
$$+\tau_{k,t} \Sigma_{J=N,T} \left[u_t^J r_{k,t}^J - \delta q_t^J - a(u_t^J) \right] \left(\check{\bar{K}}_{t-1}^J / (gn) \right) + \tau_t^{ls}$$

Fiscal Rules A: Structural Surplus Rule

- SS Rule:

$$gs_t^{rat} = gss_t^{rat} + d^{debt}\left(\check{b}_t^{rat} - \check{b}ss_t^{rat}\right) + d^{tax}\left(\frac{\check{\tau}_t - \check{\tau}_t^{pot}}{g\check{d}p_t}\right) + d^{oil}\left(\frac{\check{g}_t^X - \check{g}_{X,t}^{pot}}{g\check{d}p_t}\right)$$

- Government Surplus to GDP Ratio:

$$gs_t^{rat} = -100\frac{(B_t - B_{t-1})}{P_t gdp_t} = 100\frac{\check{\tau}_t + \check{g}_t^X - p_t^G \check{G}_t - \check{\Upsilon}_t - \frac{i_{t-1}-1}{\pi_t gn}\check{b}_{t-1}}{g\check{d}p_t}$$

- **Purpose I: Debt Stabilization**

$$gss_t^{rat} = -4\frac{\bar{\pi}_t gn - 1}{\bar{\pi}_t gn}\check{b}ss_t^{rat}$$

$$\check{b}_t = \frac{\check{b}_{t-1}}{\pi_t gn} - gs_t$$

  - To stabilize debt, must target interest inclusive deficit ratio.
  - Target debt ratio is implied by the target deficit ratio.
  - But mean reversion of debt is very slow: For 5% nominal growth rate $1/(\pi_t gn) = 0.988$.

- **Purpose II: Business Cycle Stabilization**

$$\check{\tau}_t^{pot} = \tau_{L,t} taxbase_{L,t}^{filt} + \tau_{C,t} taxbase_{C,t}^{filt} + \tau_{K,t} taxbase_{K,t}^{filt} + \bar{\tau}_{ls}$$

$$g_{X_t}^{pot} = \left(e_t p_t^{X^*,filt} \check{X}_t^{sup,filt} - \bar{d}^X \right) \left(1 - s_f^x \right)$$

  - During a boom, save excess tax and oil revenue.
  - This mainly stabilizes fiscal instruments. $\Longrightarrow$ This is a rules-based representation of "automatic stabilizers".
  - It therefore also stabilizes the business cycle relative to a balanced budget rule.

- Joint changes in multiple taxes: Need additional rules - one per instrument.

$$\tau_{c,t} = \bar{\tau}_c + d^{ctax} \left(\tau_{L,t} - \bar{\tau}_L \right)$$

$$\tau_{k,t} = \bar{\tau}_k + d^{ktax} \left(\tau_{L,t} - \bar{\tau}_L \right)$$

- Similar additional rules can endogenize government spending.

Fiscal Rules B: Explicit Output Stabilization Rule

- Output Stabilization Rule:

$$gs_t^{rat} = gss_t^{rat} + d^{gdp} \log \left(\frac{g\check{d}p_t}{gdp_{ss}} \right)$$

- Can be calibrated to OECD estimates of fiscal rules.

- With a significant positive d^{gdp} this stabilizes mainly output rather than fiscal instruments.

- But this could be very problematic:
  - We need OLG or LIQ households for a meaningful role for fiscal policy.
  - But to maximize the well-being of OLG or LIQ households policy needs to stabilize **income, not output**.

- Alternative: A more aggressively countercyclical version of the structural surplus rule (Kumhof and Laxton, 2008).

15.2 Monetary Policy

- **Taylor Rule ($k = 0$) or IFB Rule ($k > 0$):**

$$
\begin{aligned}
i_t \;=\; & E_t \left(i_{t-1}\right)^{\delta_i} \left(r_t^{filt} \pi_{4,t+k}\right)^{1-\delta_i} \left(\frac{\pi_{4,t+k}}{\bar{\pi}_t}\right)^{(1-\delta_i)\delta_\pi} \\
& \left(\frac{g\check{d}p_t^{fisher}}{g\check{d}p_t^{filt}}\right)^{(1-\delta_i)\delta_y} \left[\left(\frac{g\check{d}p_t^{fisher}}{g\check{d}p_{t-4}^{fisher}}\right)\right]^{(1-\delta_i)\delta_{ygr}} \left(\frac{\varepsilon_t}{\bar{\varepsilon}_t}\right)^{\delta_e} S_t^{int}
\end{aligned}
$$

- **Equilibrium long-run real interest rate r_t^{filt}:** Moves endogenously and permanently with savings rate shocks and other shocks (very different from Ricardian models).

- **Output gap:** Long run output $g\check{d}p_t^{filt}$ also depends on savings rate shocks.

16 Current Account and GDP

Current Account: $$e_t \check{f}_t = \frac{i^*_{t-1}\varepsilon_t(1+\xi^f_{t-1})}{\pi_t gn} e_{t-1}\check{f}_{t-1} - p^{TF}_t \check{Y}^{TF}_t - p^{DF}_t \check{Y}^{DF}_t$$
$$+p^{TH}_t \tilde{p}^{exp}_t \check{Y}^{TX}_t + \check{d}^{TM}_t + p^{DH}_t \tilde{p}^{exp}_t \check{Y}^{DX}_t + \check{d}^{DM}_t$$

International Bonds: $\Sigma_{j=1}^{\tilde{N}} \check{f}_t(j) = 0$

GDP : $$g\check{d}p_t = p^C_t \check{C}_t + p^I_t \check{I}_t + p^G_t \check{G}_t + \check{X}^x_t$$
$$+p^{TH}_t \tilde{p}^{exp}_t \Sigma_{j=2}^{\tilde{N}} \check{Y}^{TX}_t(1,j) + \check{d}^{TM}_t - p^{TF}_t \check{Y}^{TF}_t$$
$$+p^{DH}_t \tilde{p}^{exp}_t \Sigma_{j=2}^{\tilde{N}} \check{Y}^{DX}_t(1,j) + \check{d}^{DM}_t - p^{DF}_t \check{Y}^{DF}_t \ .$$

17 Model Calibration: Key Issues

- Annual version of the model for illustration purposes.
- International bonds denominated in U.S. dollars.
- Country Size: Set smallest region to 1 and others in correct proportion. Choosing shares out of 1 would cause numerical problems!
- "Big Ratios" to GDP: Use data of each region.
- Breakdown of Tax Revenue: Use data of each region.

- Long-run world real interest rate: 3% p.a., $\bar{r} = 1.03$ (by endogenizing β).
- World technology growth rate: 2% p.a., $g = 1.02$.
- World population growth rate: 1% p.a., $n = 1.01$.
- Steady state inflation rates: 2% p.a., $\bar{\pi} = 1.02$.
- Foreign exchange risk premium function: MATLAB program to pick coefficients given data for the region.

17.1 Household Sector

- 10-year planning horizon ($\theta = 0.9$): Based on empirical evidence.

$$\frac{dr^{US}}{d\left(100\frac{Government\ Debt}{GDP}\right)} = 4 \text{ basis points}$$

- 20-year average remaining working life ($\chi = 0.95$).
- Intertemporal elasticity of substitution $1/\gamma = 0.2 - 0.5$.
- Labor supply elasticity $= 0.5 - 1.0$ (by endogenizing η^{OLG}, η^{LIQ}).
- Share of liquidity constrained agents $\psi = 0.25 - 0.5$ depending on region.
- Dividend share of liquidity constrained agents $\iota = 0.125 - 0.25$.

17.2 Firm Sector

- Production function elasticities of substitution: Cobb-Douglas (1) for manufacturing, 1.5 between domestic and foreign tradables, 0.8 for nontradables/tradables, $0.5-1.0$ between government investment and consumption goods inputs, $0.5-1.0$ for oil inputs, $0.75-1.5$ between imports of different regions.

- Price markups: $10\% - 20\%$ in manufacturing and unions, $5\% - 10\%$ in distribution and retail, 2.5% for import agents.

- Quantity and price adjustment costs: To give plausible dynamics.

- Depreciation rate $\delta_K = 0.1$, $\delta_G = 0.04$.

- Government share coefficient in production function $= \alpha_G = 0.1$ (to give elasticity of GDP w.r.t. K^G of 0.14 as in the literature).

17.3 Entrepreneur Sector

- Steady state external finance premium: $1.5\% - 2.5\%$.
- Steady state leverage: $100\% - 150\%$.
- Steady state share of firms that goes bankrupt in each period: $1\% - 2\%$.
- These items are calibrated by endogenizing the structural parameters σ, μ and S^{nwd}.