Mixed First-Order Expansion of Unemployment Identity

Let $\overline{U}_t$ be the NAIRU, in percentage points, let $\overline{E}_t$ be 100 times the log of the number of persons employed, and let $\overline{L}_t$ be 100 times the log of the number of persons in the labor force. Then we have

$$\overline{U}_t = 100 - 100\frac{\exp(\overline{E}_t/100)}{\exp(\overline{L}_t/100)} \tag{1}$$

$$\iff \overline{U}_t = 100 - 100\exp\left(\frac{\overline{E}_t}{100} - \frac{\overline{L}_t}{100}\right) \tag{2}$$

We want to expand the function $\overline{U} = \overline{U}(x) = 100 - 100exp(x)$ around x^* such that $\overline{U}(x^*) = U^{ss}$. Solving for x^* we get

$$x^* = \ln\left(1 - \frac{U^{ss}}{100}\right) \tag{3}$$

Therefore, to first order,

$$\overline{U}_t = U^{ss} + \left[100 - 100exp(x)\right]'_{x=x^*}(x - x^*) \tag{4}$$

$$\iff \overline{U}_t - U^{ss} = -100exp(x^*)\left(\frac{\overline{E}_t}{100} - \frac{\overline{L}_t}{100} - \ln\left(1 - \frac{U^{ss}}{100}\right)\right) \tag{5}$$

$$\iff \overline{U}_t - U^{ss} = -100\left(1 - \frac{U^{ss}}{100}\right)\left(\frac{\overline{E}_t}{100} - \frac{\overline{L}_t}{100} - \ln\left(1 - \frac{U^{ss}}{100}\right)\right) \tag{6}$$

$$\iff \overline{U}_t - U^{ss} = -\left(1 - \frac{U^{ss}}{100}\right)\left(\overline{E}_t - \overline{L}_t - 100\ln\left(1 - \frac{U^{ss}}{100}\right)\right) \tag{7}$$