Multivariate Models of Potential Output

Economic Modeling Unit, IMF

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A Roadmap

1. A model of potential output
2. Estimation
3. Results
4. Specification in IRIS
5. Practical session

Background and motivation

- Potential output is an important variable for policy-making
- Univariate methods may miss important conditioning information (inflation, unemployment etc.)
- A mulitivariate model brings consistency between potential output and NAIRU estimates
- Estimation using Bayesian methods allows data to 'speak' (where the data have something to say)

The Model

- The Output Gap:

$$y_t = Y_t - \bar{Y}_t$$

where y_t is the output gap, Y_t is the (log) level of output, and $\bar{Y}_t$ is the (log) level of potential output.

- Operational definition of potential output: level of output that can be sustained without creating a tendency for inflation to rise or fall.

- Inflation equation:

$$\pi 4_t = \pi 4_{t-1} + \beta y_t + \Omega(y_t - y_{t-1}) + \varepsilon_t^{\pi 4} \tag{1}$$

- The unemployment gap:

$$u_t = \bar{U}_t - U_t$$

where u_t is the unemployment gap, $\bar{U}_t$ is the NAIRU, and U_t is the unemployment rate.

- Okun's law

$$u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u$$

- Relationship between the output gap and the capu gap

$$c_t = \kappa_1 c_{t-1} + \kappa_2 y_t + \varepsilon_t^c$$

- Output gap equation:

$$y_t = \rho_1 y_{t-1} - \rho_2(\pi 4_{t-1} - \pi 4_{t-1}^{LTE}) + \varepsilon_t^y$$

where $\pi 4_t$ is annual inflation in $\pi 4_t^{LTE}$ is inflation expectations:

$$\pi 4_t^{LTE} = \pi 4_{t-1}^{LTE} + \varepsilon_t^{\pi 4^{LTE}}$$

- Potential output:

$$\bar{Y}_t = \bar{Y}_{t-1} - \theta(\bar{U}_t - \bar{U}_{t-1}) - (1-\theta)(\bar{U}_{t-1} - \bar{U}_{t-20})/19 + G_t^{\bar{Y}}/4 + \varepsilon_t^{\bar{Y}} \quad (2)$$

where θ is the labor share and $G_t^{\bar{Y}}$ is an output growth trend, with

$$G_t^{\bar{Y}} = \tau G_{SS}^{\bar{Y}} + (1-\tau) G_{t-1}^{\bar{Y}} + \varepsilon_t^{G^{\bar{Y}}} \quad (3)$$

where $G_{SS}^{\bar{Y}}$ is the steady state growth rate of output

- The NAIRU:

$$\bar{U}_t = \bar{U}_{t-1} + G_t^{\bar{U}} - \omega y_{t-1} - \lambda(\bar{U}_{t-1} - U^{ss}) + \varepsilon_t^{\bar{U}}$$

where U_{ss} is the steady state NAIRU, and $G_t^{\bar{U}}$ is growth in equilibrium unemployment, with

$$G_t^{\bar{U}} = (1 - \alpha) G_{t-1}^{\bar{U}} + \varepsilon_t^{G^{\bar{U}}}$$

- Capacity utilization

$$c_t = C_t - \bar{C}_t$$

where c_t is the capacity utilization gap, C_t is capacity utilization and $\bar{C}_t$ is trend capacity utilization, with

$$\bar{C}_t = \bar{C}_{t-1} + G_t^{\bar{C}} + \varepsilon_t^{\bar{C}}$$

and growth in trend capacity utilization

$$G_t^{\bar{C}} = (1 - \delta) G_{t-1}^{\bar{C}} + \varepsilon_t^{G^{\bar{C}}}$$

Estimation

We have many unidentified parameters, making maximum likelihood estimation infeasible.

We use regularized maximum likelihood (Ljung, 1999).*

*Ljung, L., 1999, System Identification: Theory for the User, Prentice-Hall: New Jersey.

Regularized Maximum Likelihood

Let θ be a vector of parameters, let Y be the data and $L(\theta; Y)$ be the data likelihood function. Then the objective function is

$$\max_{\theta} \log L(\theta; Y) - p\Sigma_i \tau_i (\theta_i - \bar{\theta}_i)^2$$

where $\theta_i \in [\theta_i^L, \theta_i^U]$.

This method can be interpreted as a simple Bayesian technique where the prior for each parameter is a normal distribution with mean $\bar{\theta}_i$ and variance $\frac{1}{p\tau_i}$, truncated at θ_i^L from below and θ_i^U from above. The parameter estimate, θ_i^*, can be seen as the mode of the posterior distribution.

Estimation of Unobservables

- Model is linear and easily expressed in state-space, so Kalman smoother yields optimal estimates of unobservable state variables.

- *Relative* magnitudes of standard deviations of shocks are crucial - think of λ in the HP filter.

- Kalman smoother uses whole sample to derive estimates, so new observations will change history.

Steady states are important!!

For the U.S. we impose the following steady states:

Output growth = 2.5 percent

Unemployment rate = 5.5 percent

Labor share = 0.7

Current estimates

Documentation and Other work

- We have a forthcoming IMF Working Paper discussing the multivariate model where we produce estimates for 12 advanced and emerging market economies.

- We have extended the basic model to include a breakdown of potential into labor productivity and employment growth and applied it to the U.S. (see Appendix).

IRIS

IRIS is a user-friendly Matlab toolbox that:

- Makes the specification, estimation, and implementation of DSGE (and other) models easy

- Makes model simulation and forecast production easy

- Makes data management and reporting easy

  (For more details on IRIS see www.iris-toolbox.com)

IRIS

To start your IRIS session, you must first include the IRIS folder in your Matlab path, e.g:

```
addpath C:\POTENTIAL_OUTPUT_TRAINING\IRIS_Tbx_7_20090720;
```

Now, to start IRIS, type:

```
irisstartup;
```

!Important: Do not set the path via the file drop-down menu!

Example 1: Downloading data, creating time series and saving the database

Useful commands:

```
this = dbload('filename.csv')     - downloading data from .csv
this.pct_x = pct(this.x,-4)       - annual percent change
[this.tx,this.gx] = hpf(this.x)   - HP filter
this.xsa = x12(this.x)            - Census X12 seasonal adjustment
this.dx = diff(this.x,-4)         - annual difference
```

To save the IRIS database, this:

```
savestruct('filename.mat',this);
```

Example 2: Loading a database and working with time series objects

Useful commands:

```
this = loadstruct('filename.mat') - load an IRIS database
this = tseries(dates,data)        - creating a time series object
z = x{range}                      - referencing a sub-range
z(range) = [1 2 3 4]              - putting data into a sub-range
```

Using the IRIS Help Files

Help on some IRIS functions is available through Matlab's help facility. Unfortunately, limited coverage.

1. Type...

```
>> doc
```

Navigate down to the bottom of the Contents tab to *The IRIS Toolbox*

2. If you are looking for a specific function, and you know the heading under which it is classified in the IRIS manual, then type

```
>> idoc |heading|.|function|
```

For example to get help on the IRIS *get* function, type

```
>> idoc model.get
```

3. You could also type

```
>> doc |function|
```

This will take you to a disambiguation page where you can choose the function you need. If there is a Matlab function by the same name, then this way doesn't work very well (eg. *get* function).

4. Many of the IRIS functions are not yet documented in the manual. If you are comfortable using Matlab, then you can open the IRIS source code and infer the options available to you there:

```
>> edit tseries.get
```

Appendix: Labor extension

- Labor productivity (output per worker)

$$Q_t = Y_t - E_t$$

where Q_t is (log) labor productivity and E_t is the (log) number of workers employed.

- Potential output is the sum of the productivity trend and the employment trend

$$\bar{Y}_t = \bar{Q}_t + \bar{E}_t$$

- Trend labor productivity

$$\bar{Q}_t = \bar{Q}_{t-1} + G_t^{\bar{Q}}/4 + \varepsilon_t^{\bar{Q}}$$

where $G_t^{\bar{Q}}$ is a growth trend, with

$$G_t^{\bar{Q}} = \tau G_{SS}^{\bar{Q}} + (1-\tau) G_{t-1}^{\bar{Q}} + \varepsilon_t^{G^{\bar{Q}}}$$

- Decomposing employment

$$U_t = 1 - \frac{\exp(E_t)}{\exp(L_t)}$$

where L_t is the (log) labor force, with

$$L_t = N_t + P_t$$

where N_t is the (log) working age population and P_t is the (log) participation rate.

The trend labor force is

$$\bar{L}_t = N_t + \bar{P}_t$$

We want the system to be log linear, so we derive a mixed first-order expansion of the unemployment identity above.

- Unemployment trend identity

$$\bar{U}_t - U^{ss} = -\left(1 - \frac{U^{ss}}{100}\right)\left(\bar{E}_t - \bar{L}_t - 100 \ln\left(1 - \frac{U^{ss}}{100}\right)\right)$$

- Participation rate gap

$$p_t = P_t - \bar{P}_t$$

where p_t is the participation rate gap, P_t is the (log) participation rate, and $\bar{P}_t$ is the trend, with

$$\bar{P}_t = \eta 100 \ln\left(\frac{P_{SS}}{100}\right) + (1-\eta)\bar{P}_{t-1} + \varepsilon_t^{\bar{P}}$$

- Relationship between participation rate gap and unemployment gap

$$p_t = \theta_1 p_{t-1} + \frac{100}{P_{SS}} \theta_2 u_t + \varepsilon_t^p$$

- Working age population growth

$$G_t^N = \zeta G_{SS}^N + (1 - \zeta) G_{t-1}^N + \varepsilon_t^{G^N}$$

New steady states

For the U.S. we impose the following steady states:

Labour productivity growth = 1.5 percent

Working age population growth = 1.0 percent

Labor force participation = 64 percent

Unemployment rate = 5.5 percent