June 1992

# A SIMPLE MULTIVARIATE FILTER FOR THE MEASUREMENT OF POTENTIAL OUTPUT

by

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The views expressed in this paper are those of the authors.
No responsibility for them should be attributed to the Bank of Canada.

## ACKNOWLEDGMENTS

This paper grew out of work in the Model Development Group in the Research Department of the Bank of Canada. The authors wish to extend their thanks to seminar participants for many useful comments. Particularly noteworthy comments and suggestions were received from Pierre Duguay, Dave Longworth, Farid Novin, David Rose, Jack Selody and Kevin Shoom. Kevin Shoom provided excellent research assistance. David Rose contributed substantially to both the content and the exposition of this paper. The text, however, remains our responsibility.

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## ABSTRACT

This paper examines techniques that have been used to estimate potential output and finds them wanting. We suggest a simple multivariate-filtering technique that is a generalization of the Hodrick-Prescott univariate filter. In univariate filters, only information about a variable itself is used in eliminating noise in order to obtain an estimate of the underlying trend. We suggest a generalization, wherein other information is used to sharpen the identification of potential output. For example, we note that if movements in potential output have a different effect on inflation than do cyclical movements in output, then information on inflation may be useful in identifying potential output. The prospects for improving measures of potential output by using this and other information in the multivariate filter are demonstrated through Monte Carlo experiments. Evidence is also presented contrasting the results of using the multivariate filter on the historical Canadian data with the results from the Hodrick-Prescott filter and other, more traditional methods of estimating potential output. We argue that the multivariate filter has advantages over quasi-structural models of potential output because it can exploit general information from economic theory about what information might be useful, without imposing restrictions from imperfect representations of the true structure.

# RÉSUMÉ

Dans la présente étude, les auteurs analysent les techniques qui ont servi à l'estimation de la production potentielle. Les ayant trouvées déficientes, ils proposent, quant à eux, une technique de filtrage simple faisant appel à plusieurs variables, qui n'est pas autre chose qu'une généralisation du filtre à variable unique de Hodrick-Prescott. Lorsque des filtres à variable unique sont utilisés, seuls les renseignements tirés de la variable elle-même sont mis à contribution dans l'élimination du bruit et l'estimation de la tendance sous-jacente de la série chronologique. La généralisation proposée par les auteurs tire parti d'autres données pour affiner l'évaluation de la production potentielle. Les auteurs observent par exemple que si les variations de la production potentielle induisent sur l'inflation des effets différents de ceux qui découlent des fluctuations cycliques de la production, les informations obtenues sur l'inflation peuvent aider à estimer la production potentielle. À l'aide de simulations de Monte-Carlo, ils montrent qu'il est possible d'améliorer les estimations de la production potentielle en exploitant l'information relative à l'inflation et d'autres types d'information que permet de recueillir le filtre à plusieurs variables. Ils comparent aussi les résultats qu'ils obtiennent en appliquant leur technique aux séries chronologiques canadiennes pour estimer la production potentielle avec ceux qui sont obtenus à l'aide du filtre de Hodrick-Prescott et de méthodes plus traditionnelles. Les auteurs soutiennent que la technique de filtrage à plusieurs variables est supérieure aux modèles quasi structurels ayant trait à la production potentielle en ce sens qu'elle permet, à partir de renseignements généraux tirés de la théorie économique, de repérer des informations utiles sans imposer de restrictions axées sur des représentations imparfaites de la structure véritable de l'économie.

# 1 INTRODUCTION

The use of a macroeconomic model for forecasting and policy analysis requires simple and effective methods of estimating potential output.[1] In most models, the gap between aggregate demand and potential output (that is, the level of excess demand or the output gap) is the key driving variable in the nominal (wage and price) dynamics. Understanding where the economy is going requires a good idea of where it is now; this is more difficult than it may seem. The basic problem is that potential output and hence the output gap are not directly observable. They must be derived from their hypothesized determinants and other information, such as observable variables that are thought to be correlated with the desired measures. This has made econometric modelling of potential output quite difficult.

The accuracy of measures of potential output is particularly important for monetary and fiscal policy. In the case of monetary policy, for example, the task is often characterized as one of setting a path for short-term instruments (such as the nominal interest rate or the exchange rate) in order to produce the output gaps necessary to achieve an ultimate nominal target (such as zero inflation) by some date in the future. However, if potential output is mismeasured to the point, say, of calculating a negative output gap when in fact it is positive, then monetary policy could amplify the business cycle and, indeed, could fail to achieve its ultimate goal. In a similar vein, one might argue that the fiscal problems faced by so many industrialized countries may be a consequence of fiscal authorities' overly optimistic assumptions about potential output growth and the tax base that goes along with it.

Errors in estimating potential output have resulted in monetary and fiscal policy errors (Freedman, 1989). Indeed, not only were estimates of potential output for the 1970s overly optimistic, but revisions were made only after several years of observable errors (Gordon, 1979). Thus, the possibility

1. The view on potential output in this paper is taken from the perspective of a monetary authority. Thus, potential output is defined as the maximum level of output that can be produced without creating pressure for inflation to rise. This is the same definition that was used by Okun (1962).

exists of making meaningful gains by improving measures of potential output.

This paper introduces a method that promises improved estimates of potential output. The method is a simple extension of the Hodrick-Prescott (1980) filter.[2] The H-P filter is extended by adding information about prices and unemployment to corroborate the evidence from univariate estimates of potential output. In this way, the multivariate filter occupies a middle ground between univariate measures of potential output – whether simple time trends or more complex filters, such as moving averages -- and full, simultaneous estimations of potential output along with the processes generating inflation and unemployment, such as in Ford and Rose (1989), Adams and Coe (1990) and Kuttner (1991).[3]

The remainder of this paper is organized as follows. In Section 2, we provide a summary of methods that have been used to estimate potential output. We do not have the space to be exhaustive in our review, so a topical discussion aimed at bringing out the broad methodological issues is offered instead. Section 3 introduces the multivariate filter and provides measures of potential output for Canada. In Section 4, we turn to Monte Carlo methods to provide estimates of the gains in efficiency that might be expected from using the multivariate filter. The final section adds some concluding remarks and suggests some possible extensions.

We conclude that the multivariate filter produces substantially more reliable (that is, lower variance) estimates than univariate methods, but that there remains a significant amount of uncertainty in even these improved estimates. We argue that this uncertainty can account for an important part of the overall uncertainty in simulations of macroeconomic models. A major conclusion is that policy advice that depends in any significant way on the presumed course of potential output should consider explicitly the uncertainty in the measures themselves.

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2. A computer program that implements the multivariate filter in RATS is available from the authors.

3. Kuttner, whose work came to our attention after this research had been completed, is motivated by concerns very similar to our own and comes to similar conclusions.

## 2 THE MEASUREMENT OF POTENTIAL OUTPUT: A HISTORICAL PERSPECTIVE

The history of measuring potential output mirrors economic thought regarding the importance of the supply side of the economy and economists' understanding of it.

In the 1960s and early 1970s, the accepted methodology for evaluating potential output was quite simple: estimates were derived by passing time trends through the peaks of the business cycle. Since macroeconomic time series showed growth over time, something had to represent this growth, and time trends were the obvious candidate.

The time-trend era coincides with the era when macroeconomic models were primarily Keynesian expenditure systems. The supply side of the economy was regarded as deterministic and unimportant in business cycle analysis. The focus on the peaks of cycles reflected the notion that potential output should be defined as the maximum possible output. The through-the-peaks method embodied a mixture of engineering notions of physical capacity and a belief that the unbridled economy tends towards inefficient outcomes (that is, outcomes where output is below potential output). By construction, this method resulted in output gaps – that is, in differences between actual and potential output -- that were almost always negative[4] (Figure 1, p. 28). Many economists thought that closing such gaps was an appropriate goal of policy.

Through most of the 1960s, output growth was smooth and inflation was stable, and the conceptual problem with persistently negative output gaps remained dormant. This permitted most applied researchers to ignore the conceptual link between the output gap and inflation that had been clearly identified in the seminal work of Okun (1962). However, when policy makers responded to shocks in the 1970s by attempting to achieve output goals that turned out to be unrealistic, difficulties began to surface. Insofar as

4. Because of the focus on how far below the maximum output an economy was, these gaps were usually defined the other way around, as potential minus actual output (that is, positive numbers). Our perspective in the text reflects the current convention of defining gaps for use in the analysis of inflation dynamics.

such gaps were used to explain changes in inflation, by means of a Phillips curve, for example, the through-the-peaks measure was not helpful. A line linking peaks in output will not provide reasonable estimates of economic potential growth between the points, unless the extent of excess demand is about the same at all cycle extrema.[5] There was also a severe problem at the end of samples, where it was often unclear what trend line was appropriate.

The economic logic of defining potential output with reference to the link between excess demand and inflation, which came to the fore in the debates about the natural-rate hypothesis in the late 1960s and early 1970s, spelled the end of the through-the-peaks method of measuring potential output.[6] It was replaced by linear time trends calculated to run roughly through the centre of the business cycle. This was an advance in that all observations were used in estimating the underlying trend, rather than just the peaks. However, the trends were generally estimated on the presumption that output was at potential, on average, over the sample. In an inflationary period, this presents a problem.

By the mid 1970s, the pure time-trend approach was being abandoned because it had produced unsatisfactory estimates of potential output (see, for example, Perloff and Wachter, 1979). In retrospect, it is quite clear that no single linear segment could have "explained" in any satisfactory way the trend in output much less the equilibrium level of output; there was too much variation in output and inflation.

Attention began to turn towards supply-side influences. The profession's initial response to these "supply" shocks was simply to "correct" for them. In some instances, regressions of output on time along with such things as spline functions and shift dummies were used, in others the "eyeball metric" was preferred.[7] Generally speaking, however, the old view remained

5. One could add a constant shift parameter in regressions using such a measure, but this does not remove the fundamental problem.

6. Nevertheless, there are still proponents of the earlier method. See De Long and Summers (1988).

7. In a Bank of Canada research memo, Gosselin et al. (1981) use this colourful description of the method for specifying total factor productivity at the time: "To put this all very starkly, given the data and research, both a sharp eye and a flexible 18-inch ruler were applied to the data in a manner consistent with existing priors." The statement would have been no less accurate a depiction of the methods used to detrend aggregate output. This approach was by no means unique to the Bank, although few others were so clear about it.

that, with rare exceptions, the supply side of the economy was a smoothly evolving, if ill-understood, process, and potential output continued to be inferred from aggregate output alone.

As the 1970s progressed, it became clear that even a continuous updating of a regression of output on time would have resulted in significant and unpalatable changes in the implied output gaps. Developments such as the oil price shocks and the productivity slowdown showed that supply shocks could be important. It is now widely accepted that both demand and supply shocks can have important influences on variations in output (Boschen and Mills, 1990).

The next step in the progression was one of disaggregation. There was an examination of the contributions to economic growth of individual industries (for example, Denison, 1985). This increased understanding of what had happened to the structure of the economy, but it was of little help to macroeconomists, in part owing to the enormous data requirements of such studies, which make them impractical for ongoing current analysis. Of more practical importance were the efforts at disaggregation by factor inputs, following the lead of Solow (1957). There was hope for this method because it was felt that the effects of such things as oil price shocks could be estimated.[8]

There are usually two steps in this approach. First, a production function is specified. This provides a decomposition of output into portions that are explainable by inputs like capital and labour as well as by the "Solow residual," or total factor productivity. Solow had found that only about one-third of growth could be explained by factor inputs, with the rest left for the residual. A wide variety of production functions have been employed for the purpose of reducing the residual, including the Cobb-Douglas and the CES specifications, as well as a variety of more flexible forms, such as the translog function. For heuristic purposes, we use a Cobb-Douglas produc-

8. Stuber (1986) stresses the impact of oil prices in his survey of explanations for the productivity slowdown in Canada. SAM (Rose and Selody, 1985) and MACE (Helliwell et al., 1982) are small- and large-scale examples of models with fairly elaborate supply sides constructed around the importance of oil prices.

tion function with two inputs, capital and labour, under constant returns to scale. In logarithms, this is written:[9]

$$Y_t = \alpha_t + \beta K_t + (1-\beta) L_t \tag{1}$$

where $Y$ is output, $\alpha$ is total factor productivity (TFP), $\beta$ represents capital's share of income, $K$ is capital and $L$ is labour.[10] An estimate of $\beta$ can be obtained from econometric methods or, in this case, from simple historical averages of the income shares of capital and labour. With such an estimate and time-series data for output and the inputs, a residual time series for TFP, the $\alpha_t$ in equation (1), can be computed.

The second step involves obtaining trend or "equilibrium" estimates for the factor inputs and for TFP.[11] Potential output is sometimes taken as that level of output that is consistent with the *trend* or *equilibrium* levels of the contributing inputs and productivity. Usually, however, economists define potential output using the actual stock of capital, arguing that it is this measure that is appropriate in gauging the extent of excess demand and inflationary pressure.

One important advantage of the production-function approach over the time-trend approach is that it is easy to keep track of the major contributing factors to potential output. That is, variations in potential output can be decomposed into variations in its underlying determinants such as labour

9. The Cobb-Douglas specification remains popular for macroeconomic models because of its simplicity, the desire for which reflects the lack of success model builders have had with more complicated models of the supply side of the economy. The profession's inability to explain potential output at the macroeconomic level has spawned some impressive efforts at disaggregation, but these have run into difficulties with data availability and have tended to be too unwieldy for use in macroeconomic models. More fundamentally, these complicated supply-side structures have tended to focus on correcting for the last source of a supply shock rather than treating supply shocks as a stochastic process coming from a multitude of sources.

10. Labour input is usually decomposed into changes in population, labour force participation, unemployment and average weekly hours worked. Attention has also been given to labour quality, and the effects of things like education levels and spending on research and development. Since such issues are of no consequence to us here, we ignore them.

11. In some studies there is an explicit adjustment for capital utilization to put capital inputs into effective units. With Cobb-Douglas technology this does not add anything unless reliable independent estimates of capacity utilization can be obtained from other methods. The methods that have been used to develop these estimates suffer from even more severe problems than those we discuss in the text. See Schaefer (1980) and Shapiro (1989).

force growth, capital formation and trend TFP.[12] For this reason the production-function approach was seen as an attractive framework for organizing the data. However, there remained a problem. Since economists had no useful model of the determinants of productivity, the resulting estimates of potential output were still essentially exogenous time trends.

An exchange between conference participants at a 1978 session of the Carnegie-Rochester Conference Series "Aspects of Policy and Policymaking" exemplifies the debates on measuring potential output at that time. Perloff and Wachter (1979) had constructed measures of potential output using a translog production function with three factor inputs: capital, labour and energy. Their production function included a polynomial time trend and elasticities that were allowed to vary according to the gap between the unemployment rate and the non-accelerating-inflation rate of unemployment (NAIRU). Alternative measures of the NAIRU were provided by calculating the trend unemployment rate corrected for demographic shifts, by computing the prediction of an estimated unemployment equation and by inverting an estimated wage equation. The last of these three methods produced the highest estimates of the NAIRU. Perloff and Wachter (P-W) used the first method to calculate the unemployment gap, which they then used in a regression on a third-order polynomial time trend and a constant to construct a series for potential labour inputs. They argued that their results compared favourably with those of their predecessors because the estimated output gap for the United States in 1977Q4 was smaller and because their measures did not come from the trend-through-peaks method.[13]

The discussants of P-W's work raised a number of issues. Tatom (1979) argued that the demographic-based measure of the natural rate of unemployment ignored the fact that the purpose of calculating the NAIRU was to predict levels of aggregate demand beyond which inflation should be affected. Thus, although the P-W potential output series was not through the peaks of output, it had no direct link to the concept of a NAIRU. Picking

12. When it is necessary to provide forecasts of potential growth, such a decomposition makes it easy to construct different scenarios under various assumptions about the components.

13. P-W's work is a good example of this approach, but the same ideas were used by many researchers. For example, Tatom (1982), Perry (1977), and Clark (1977) use essentially the same methods.

up on this theme, Gordon (1979) argued that potential output should be redubbed "natural output" to reflect the economic nature of the idea. It was Plosser and Schwert (1979), however, who made the following important observation. For all the sophistication of the production functions that were specified, it was the polynomial time trends that provided the "explanatory power" in the regressions.

It was the continued use of polynomial time trends as regressors, together with their quantitative importance to the results, that was the telling weakness of the production-function approach. In fact, the production-function approach required the same sort of ad hoc fix-ups as the earlier, pure time-trend estimates. Since potential output was taken from the fitted values of the estimated production function, the purpose of these regressions was, in effect, to produce autocorrelated residuals which could be taken as measures of excess demand. The results of P-W's work indicate as much, in that the Durbin-Watson statistics range as low as 0.2. This was a common feature of such work. In the end, as Plosser and Schwert (1979, p. 185) note: "most efforts to estimate potential output ... are essentially equivalent to trend extrapolation of output."[14]

Finally, one might also note that the range of estimates provided by P-W and their contemporaries was very large, thereby providing little in the way of direction for monetary policy. Moreover, while there was much comparison of point estimates, little attention was paid to the wide range of values obtained and the subtle, interpretative issues that gave rise to those differences in estimates. It would turn out that nearly all estimates of potential growth in the 1970s were too high.[15]

14. Nelson and Kang (1984) would later demonstrate that these sorts of regression results with high t-statistics and low Durbin-Watson statistics are likely to be completely spurious.

15. Perry (1971) provided projections for potential output growth for the U.S. economy over the 1970s. He wrote: "On these assumptions, potential output grows at a rate of between 4.2 and 4.4 percent each year in the 1970's. For the decade, potential output growth averages 4.3 percent annually, an indisputable record for any decade in recent history. If the price deflator rises at an annual rate of 2.5 percent over this period, potential GNP in current prices would reach $2 trillion by the end of the seventies." Real GNE growth in the United States averaged 2.8 per cent in the 1970s. If we assume that output growth was, on average, equal to actual output growth, Perry's level error by the end of the 1970s would also have been around 20 per cent. In 1970, the Economic Council of Canada predicted that labour productivity (GDP/Labour Force) in Canada would grow by 3.0 per cent per year between 1970 and 1980. It actually grew by 1.3 per cent, resulting in a level error of about 20 per cent by 1980. The projections for other countries included in the Economic Council's study were even larger in some cases. Similarly over-optimistic projections of potential can be found in OECD publications of the same era.

To summarize, objections to the techniques used in the 1960s and 1970s to measure potential output stemmed from three conceptual difficulties. First, there were only very indistinct links between the purpose for which potential output was being calculated and the methods for doing so. Insofar as potential output is used to predict inflation, it seems logical that it be constructed with that intention in mind. Second, there was little sense of what constituted an "explanatory variable" in a structural sense. The emphasis was on searching over a list of supposedly exogenous variables in order to find a good fit, with coefficients and implied output gaps that were not at odds with the researcher's a priori beliefs. Third, the connection between the difficulty of finding these exogenous explanatory variables for structural estimation and the uncertainty regarding those estimates was given little thought. Notwithstanding the poor quality of much of the data and the primitive state of economic knowledge of the determinants of the supply side of the economy, very little attention was given to the lack of precision in the estimates.

Two new initiatives emerged in the 1980s. The first approach -- let us call it the *structural approach* -- entailed a search for better measures of the structural shifts in potential output and the NAIRU, rather than the usual ad hoc shift dummies. For example, Lilien (1982) proposed sectoral dispersion variables to pick up the influence of "matching" problems in the labour market. In addition, there was an attempt to exploit more powerful econometric techniques. Rose (1988) and Coe (1990) searched for demographic and policy variables that could explain cyclical and secular movements in the NAIRU. Given the fact that most macroeconomic shocks affect prices, wages, unemployment and output simultaneously, Clark (1983), Ford and Rose (1989) and Adams and Coe (1990) used cross-equation restrictions to examine the joint determination of these variables and to obtain consistent estimates of potential output.

The second strand of literature -- let us call it the *stochastic* approach -- treated disturbances to the NAIRU and to potential output as stochastic phenomena.[16] To implement this approach, identifying restrictions were

16. The focus on stochastic trends in macroeconomic time series probably began with Beveridge and Nelson (1981) and Nelson and Plosser (1982). A vast body of literature now exists in this area. Useful references include Watson (1986), and Stock and Watson (1988).

imposed to help separate the demand shocks from the supply shocks. For example, one set of identifying assumptions that has been used quite widely maintains that supply shocks have permanent effects on output while demand shocks have only temporary effects. Simple univariate filters, such as the Hodrick-Prescott (H-P) filter, have been used to estimate the permanent component.

In some applications, the structural and stochastic detrending approaches have been combined. For example, Ford and Rose (1989) use the H-P technique to estimate the permanent component of total factor productivity and then use these trend TFP estimates to develop econometric estimates of equilibrium labour inputs. The H-P filter is also currently used to estimate trend TFP in RDXF, the Bank of Canada's quarterly forecasting model. The top right-hand panel of Figure 2 (p. 29) shows the measure of TFP implied by RDXF's Cobb-Douglas representation of commercial output.[17] This panel also shows the trend measures produced by the H-P filter; the resulting "TFP gaps" are shown in the bottom panel. The top left-hand panel reports the estimates of potential output derived when these trend TFP assumptions are combined with RDXF's measure of trend labour inputs.

There are several interesting observations to be gleaned from Figure 2. First, the top right-hand panel shows that growth in the capital stock and the labour force are not the only sources of secular trend in output. Total factor productivity also shows growth over time, although not at anything close to a constant rate. Moreover, there appears to be an important cyclical pattern to TFP. This figure makes it clear why structural estimates of potential output are forced to rely on flexible time trends and why the H-P filter has gained popularity. We prefer the H-P detrending technique to the use of polynomial time trends, because the H-P technique will have better properties if there is an underlying stochastic trend in the series. However, we will show in the following two sections that univariate detrending methods, like the H-P filter, should not be expected to produce reliable estimates of the

17. Total factor productivity in Figure 2 is defined from a Cobb-Douglas production function evaluated for commercial output and actual labour and capital. The *trend* total factor productivity series of RDXF is derived by using the H-P filter, as shown in Figure 2, but a small amount of judgment is applied as well.

underlying trend component of aggregate output. Figure 2 illustrates that the production-function approach will suffer from the same sorts of problems if the detrending method is applied to an important component, such as TFP.

We would argue that there is insufficient knowledge about the true structural determinants of the supply side of the economy to make the structural approach practicable. Attempts to model markets in a stochastic setting have provided interesting insights into how the economy works, but these models are far too simplistic to implement empirically. This has meant that macroeconomists have tended to use highly stylized models that focus on the specific shocks that are easy to identify. Although these models have succeeded in obtaining estimates of how the economy might respond to, say, changes in energy prices or in the unemployment insurance (UI) regime, they have not provided a reliable guide in estimating potential output. Indeed, there has been a tendency to cling too long to obsolete estimates of potential or to over-attribute apparent changes in potential output (or the natural rate of unemployment) to shocks that are easy to identify.

Even in cases where shocks have been easy to identify, the structural approach has produced an incredible range of estimates. For example, estimates of the effects of the 1971 UI revisions on the NAIRU, based on time-series regressions, range from no effect to 2 percentage points (see Rose, 1988, for a summary). The range is even larger in estimates of the output effects of the energy price shocks in the 1970s.[18] We think that this provides a good measure of how flimsy these estimates really are and of the importance of the whimsical modelling decisions that had to be made to produce them.

One might ask whether matters have improved in recent years. By way of example, if we were to experience a series of shocks similar to those of the 1970s, would policy makers be better able to avoid the mistakes that

18. One example of a variable often used in this way is the price of oil. It should be pointed out that, in Canada, every major oil price shock has been accompanied by important fiscal and regulatory changes. Unless one models the political decision-making process, the likelihood of obtaining the same combination of effects in the future is slight. Note also that such large disruptive shocks might be expected to induce industry to invest in new, more flexible and more energy-efficient technology. This, too, limits the scope for structural modelling.

allowed inflation to accelerate during those years? In spite of our critical review of the history of measuring potential output, we believe that the answer to this question is "yes." With the lessons of stagflation in mind, policy makers have become more sceptical about all measures of potential output and more aware of the importance of supply considerations generally for policy analysis. Policy makers have also become more aware of the importance of preventing inflationary shocks from triggering an escalation of inflation expectations. If it is concluded that the source is a demand shock, policy makers are more likely to react strongly and persistently than has been the case in the past.

In effect, policy makers and their advisers have adapted to the stochastic environment in which they have found themselves. There is a tendency to use much more judgment and more structured, but less detailed macroeconomic models.[19] This is a by-product of the realization that, although economists have a grasp of the broad relationships between macroeconomic activity and such unobservables as potential output, the structural form of these relationships remains elusive. This would seem to call for a method of measuring potential output that reflects general beliefs concerning the nature of these relationships, without imposing too much formal structure.

As we stated in the introduction, the use and maintenance of a macroeconomic model for forecasting and policy analysis requires simple and effective methods for estimating potential output. Methods used in the past have not met these criteria. We propose a multivariate filter that exploits the information economists already rely on in arriving at judgments about potential output. The multivariate filter can be thought of as a partially structural approach, in that it looks to economic theory to provide sources of information, but does not impose the restrictions of any particular theory. In this sense, it occupies a middle-ground between full simultaneous equations estimation and ad hoc time trend methods.

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19. It would be difficult to incorporate into a formal econometric model all the information typically used by economists to infer the source of shocks to potential output. Moreover, even if an attempt were made to do this, the model would likely be so complicated that, for most purposes, it would be very difficult to maintain. Today, it is frequently the case that macroeconomic models focus on aggregate dynamics and consistency and leave to highly specialized models the task of analyzing the effects of specific supply shocks that are easy to identify.

## 3 PROPERTIES OF FILTERS

It is our view that potential output is best characterized as being driven by a stochastic process. That is, in addition to the myriad of policy, demographic and commodity price shocks that have been proposed as having precipitated movements in potential output, there are regular unidentifiable shocks, some of which have long-lasting effects on potential output. Without being able to identify these shocks, economists face significant difficulties in measuring potential output on a quarter-to-quarter basis. If output surges ahead this quarter, is this to be taken as excess demand or a change in potential? On what basis should one decide?

There can be little doubt that structural estimation can provide some answers. In the presence of model uncertainty, however, acting on a rigid structural view can lead to costly policy errors, if that structural view turns out to be wrong. It is the difficulty of finding sufficient evidence to arrive at a consensus on structure that has spawned astructural methods of measurement, such as the H-P filter, at least as complementary tools. The H-P filter has been used extensively by proponents of real-business-cycle models.[20] It is also interesting to examine the H-P filter because it encompasses the deterministic time-trend model as a special case, and because our suggested multivariate filter can be seen as a generalization of the H-P filter.

### The Hodrick-Prescott Filter

The rationale for using the H-P filter is that it can help decompose an observed shock into a permanent (supply) and a temporary (demand) component. For a univariate filter, the only identifiable difference between supply shocks and demand shocks is that the former have permanent effects on the real variable in question, while the latter have only temporary effects. However, if the temporary component contains a great deal of persistence, it is very difficult to distinguish between the two, particularly at

20. Detrended output, using the H-P filter, is not explicitly considered to be potential output in real-business-cycle models. Singleton (1988, p. 361) puts it this way "it is often assumed that business cycle models are designed to explain cyclical phenomena.... Accordingly, secular variation is often removed from economic time series..." through use of the H-P filter. The sole distinction of importance here is that the deviation of output from its secular component is not taken to be an "output gap" as is the case in Keynesian models.

the end of a sample. In fact, the categorization of permanent shocks as supply shocks (and vice versa) and temporary shocks as demand shocks can be misleading, since there is no reason to believe, for example, that supply shocks cannot be temporary.[21] Similarly, Lilien (1982) describes a plausible mechanism by which what would be considered demand shocks can have important, albeit temporary, effects on aggregate supply. Furthermore, as King and Rebelo (1989) emphasize, the two sources of shocks need not be independent; in endogenous growth models, cycles and growth are part of the same phenomena. Nevertheless, economists realize that supply shocks and demand shocks have inherently different implications for inflation. Therefore, to analyze the inflation process, we need some identifying restriction to disentangle demand shocks from supply shocks.

The H-P filter is derived by minimizing the sum of the squared deviations of a variable, $y_t$, from its trend $\tau_t$, subject to a "smoothness" constraint that penalizes squared variations in the growth of the trend series. That is, the H-P trend values are those that minimize:

$$\mathfrak{I} = \sum_{t=1}^{T} (y_t - \tau_{y,t})^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{y,t+1} - \tau_{y,t}) - (\tau_{y,t} - \tau_{y,t-1})]^2 \qquad (2)$$

The result is an equation where $\tau_{y,t}$ is a function of $\lambda$ and of both past and future values of $y_t$. The properties of the H-P filter are reviewed in some detail by King and Rebelo (1989), who note that for some specifications of the data-generating process of permanent and temporary shocks, the H-P filter is an optimal linear inverse filter.[22] It is quite general, in that it can render stationary any time series that is integrated up to fourth order. In the middle of a sample, the H-P filter is a symmetric, two-sided filter. The two-sidedness removes the problem of in-sample phase shift but becomes a problem at the end of the sample, since future data are not available.

The user can determine the smoothness in the trend series by choosing an appropriate value for $\lambda$, the "smoothness" parameter. Higher values of $\lambda$

21. The somewhat misleading delineation of permanent shocks as being supply shocks and temporary shocks as being demand shocks originates -- at least in its most recent variant -- from the identifying restriction used in Blanchard and Quah (1989), where it was used to determine how prevalent "supply" shocks are. See also Dea and Ng (1990).

22. "Optimal" means minimum-variance estimator of the permanent component.

imply a larger weight on smoothness in the trend series. In the limit, as $\lambda$ becomes arbitrarily large, the trend series will converge on a linear time trend. A very small value of $\lambda$ will effectively eliminate the penalty function and the $\tau_{y,t}$ series will be set equal to the actual series.[23]

King and Rebelo show that for a common decomposition of time series into permanent and transitory shocks, the optimal value of $\lambda$ will be a function of the ratio of variances of those shocks. There is, however, considerable debate concerning the relative variance of supply and demand shocks (see Eichenbaum, 1990). In practice, users have typically followed Hodrick and Prescott and set $\lambda = 1600$ for quarterly data (as we did in Section 2). Kydland and Prescott (1990, p. 9) provide the following rationale for this choice:

> We found that if the time series is quarterly, a value of $\lambda = 1600$ is reasonable. With this value the implied trend path for the logarithm of real GNP is close to the one that students of business cycles and growth would draw through a plot of this series.

Figure 3 (p. 30) provides plots of trend real GDP for $\lambda = 1600$ and for a very large $\lambda$. The deviation of output from potential for the two cases is shown in the bottom panel. The case of a very large $\lambda$ in the top-left panel -- which is effectively a time trend -- is an interesting special case, because it closely approximates the standard view held in the 1960s and 1970s. That is, it assumes that the variance of supply shocks is very small relative to the variance of demand shocks.

The level of the gap produced by the simple detrending method is clearly not consistent with the traditional equilibrium notion of potential output. Moreover, the position of the simple trend will depend a lot on the sample period chosen. The trend line in the top-left panel of Figure 3 uses the data from 1953 to 1990. One would obviously obtain quite different estimates of the trend and the output gaps if one estimated the trend on the data to the mid-1970s, excluding the period of slower productivity growth since then.

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23. Hence it can be said that the H-P filter nests the extreme Keynesian and real-business-cycle models within its set of parameter choices. Choosing a large $\lambda$ imposes the view that supply shocks are deterministic and that variations in output come almost entirely from demand shocks. Choosing $\lambda$ to be very small imposes the view that most variations in output are also variations in potential or trend output and hence are driven by supply shocks.

A careful examination of Figure 3 shows that for the large-λ case, the change in inflation appears to be more closely related to the change in the output gap rather than its level. It is easy to show that this can be a false conclusion if the level of the gap is mismeasured. If potential output contains a stochastic component with substantial persistence, then including the change in the gap in a regression will tend to yield significant estimated effects from the change in the gap, when no such effect is truly there.[24] It is quite common to find empirical estimates of the Phillips curves with "significant" coefficients on the change in the gap.[25]

Now consider the case of $\lambda = 1600$. For this choice, the estimates of the stochastic trend component tend to follow the actual series much more closely. There is a tendency to attribute any dramatic change in the series to both supply and demand influences. Even if this attribution is correct *on average*, however, the technique will understate the degree of excess demand in cases of pure demand shocks. For example, when the monetary conditions are being tightened to bring about a deceleration in inflation, the H-P filter will tend to understate the amount of excess supply in the economy. Indeed, even if one knew the true relative variance of supply shocks, and therefore the optimal value of λ, there would still be considerable uncertainty in the estimates. Furthermore, the level of uncertainty in the estimates will be greater at the end of the sample, where it matters the most. The reason for this is quite simple and it is not specific to the H-P filter. The only identifiable difference between supply shocks and demand shocks for a univariate filter is that the former have permanent effects on the real variable in question, while the latter have only temporary effects. If the effects

24. See Laxton, Shoom and Tetlow (1992) for evidence of this point.

25. This was not always the case. Indeed, Gordon (1980) argued that the level-gap specification "obscures the fact that price change has been much more closely related to the contemporaneous rate of change of detrended output" and that the literature "has shown no awareness of the importance of the ROC [rate-of-change] phenomenon." Our Monte Carlo experiments have shown that even if the true relationship is from the lagged level of excess demand to inflation, the coefficient on the change in the lagged gap will tend to be significant and even larger than the coefficient on the level gap. This is also true for less extreme values of λ. Our point here is not that one model should be preferred to another. Our point is that one has to take the measurement of the gaps very seriously in any study of inflation dynamics. In principle, potential output should be derived in a manner that is consistent with the structure of the model that is being investigated. Furthermore, the dynamic specifications should be based on the merits of the theoretical arguments, rather than empirical estimates of reduced-form equations that contain important measurement errors.

of demand shocks are very persistent, it will be very difficult to distinguish between permanent and temporary shocks. This problem will be even more severe at the end of the sample, because almost no information is available about the long-term effects of the latest shocks.

### The Multivariate Filter

We have argued that one should place very large confidence intervals around estimates of potential output that are derived from univariate filters or polynomial time trends. We have also argued that estimates derived using these techniques will be even more unreliable at the end of a sample. This is a major disadvantage, because the most recent conditions are precisely those needed for projections and policy analysis. An important objective of the multivariate filter is to reduce the level of uncertainty associated with the estimates of potential output.

In order to attribute the proportion of a given shock that can be deemed as originating from a supply disturbance, we rely on two well-established empirical relationships, the output-inflation relationship and the output-unemployment relationship. We write these as equations (3) and (4):

$$\pi_t = \pi_t^e + B(L)\,(y_{t-1} - \tau_{y,t-1}) + \varepsilon_{\pi,t} \tag{3}$$

$$U_t - \tau_{U,t} = C(L)\,(U_{t-1} - \tau_{U,t-1}) + D(L)\,(y_{t-1} - \tau_{y,t-1}) + \varepsilon_{U,t} \tag{4}$$

where $\pi$ is the inflation rate, $\pi^e$ is the expected inflation rate, $y$ is the log of output, $U$ is unemployment, $\tau_{i,t}$ is the trend value for variable $i$, and the $J(L)$ are polynomial lag operators.[26] Equation (3) is a Phillips curve, wherein inflation today is determined by expected inflation and the history of output gaps. Equation (4) is an Okun's Law relationship, which maps output gaps into unemployment gaps. Frequently, (3) is augmented with an autoregressive representation of expected inflation:

$$\pi_t^e = A(L)\,\pi_{t-1} \tag{5}$$

26. That is, $A(L)z_t = (a_0 + a_1 L + a_2 L^2 + \ldots)z = a_0 z_t + a_1 z_{t-1} + a_2 z_{t-2} + \ldots$, and analogously for the polynomials expressed as a function of $z_{t-1}$ in the text. See Sargent (1987, pp. 176-183) for a review of the algebra of lag operators.

with the so-called accelerationist restriction $A(0) = 1$ imposed.[27] We do not take any of these three relationships as being structural in the usual sense. For the purposes of this paper, it is sufficient that there be broad agreement that *information* regarding inflation and unemployment can be represented by equations of this form. The two error terms, the εs, are included to reflect specification errors.

Since equations (3) and (4) contain three unobservable variables $\tau_j = \{\tau_y, \pi^e, \tau_U\}$, there are some difficult econometric issues regarding consistency and exogeneity. It has been common in econometric work to ignore these issues by using estimates for these unobservable variables from other sources. It is not our intention to bridge this gap. Rather, we regard our multivariate filter as a complement to full, structural estimation of $\tau_j$. The generalized problem is to minimize:

$$\Im = \sum_{t=1}^{T} \eta_t \varepsilon_{y,t}^2 + \sum_{t=1}^{T} \theta_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^{T} \gamma_t \varepsilon_{U,t}^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{y,t+1} - \tau_{y,t}) - (\tau_{y,t} - \tau_{y,t-1})]^2 \qquad (6)$$

subject to (3)-(5). The vectors $\{\eta, \theta, \gamma\}$ represent the weights that are attached to the series of gap terms $\varepsilon_j = \{\varepsilon_y, \varepsilon_\pi, \varepsilon_U\}$ at each point in time. The last term in the optimization problem is just the H-P penalty function. Note that we are taking $\tau_U$ as given. This need not be the case, in principle. The minimization problem could be generalized to determine $\tau_y$ and $\tau_U$ simultaneously.

The H-P optimization problem has been generalized in two ways. First, we have allowed for time-varying weights on the three gap terms. This option is included to allow users to incorporate information from other sources.[28] The second and more important extension is that information about the output-inflation process and the output-unemployment process has been

27. Sargent (1971) has shown that this restriction is not necessary for the expectations-augmented Phillips curve to be truly "accelerationist." Since we are not concerned with inference here, we put this issue aside.

28. For example, users might wish to exercise judgment regarding periods of time when one of the relationships might be regarded as unreliable, such as the inflation equation during times of wage and price controls. In addition, the weights provide an easy way of benchmarking excess demand at certain dates. There has been a long tradition of using detailed information to benchmark potential output to actual output in quarters where excess demand is believed to be close to zero. This can be done quite simply in the filter by setting a high value for $\eta$ for that quarter.

included in the optimization problem. By selecting positive values for $\theta$ and $\gamma$, estimates of potential output will be chosen in a way that helps to improve the fit of the inflation-rate equation and the unemployment-rate equation. If there is useful explanatory power in these equations, then the data on inflation and unemployment will help in the identification of potential output.

There are a number of ways in which one might wish to choose the weighting factors -- $\eta$, $\theta$, $\gamma$ and $\lambda$. One method would be to base them on some measure of the relative uncertainty in the relationships. For example, if the Phillips curve had a relatively high error variance, one might want to use a relatively smaller value for $\theta$. Another would exploit the frequency of data revisions, particularly for the weights at the end of the sample.[29] Also, as noted earlier, one could exploit explicit information or judgments to change the weights in particular periods. We first illustrate the multivariate filter using fixed, equal weights on the squared errors and the standard choice of 1600 for $\lambda$. For future reference, we index this choice of weights as $MV(\eta,\theta,\gamma,\lambda) = MV(1,1,1,1600)$.

The results are shown in Figure 4 (p. 31). To facilitate comparison, the pure H-P filter case, MV(1,0,0,1600), is also shown. It would appear that the H-P filter, by following actual output downward in the 1981-82 recession, underpredicts the extent of disinflation in 1983. In addition, the H-P filter largely misses the excess demand associated with the acceleration of inflation between 1970 and 1974. By comparison, the multivariate filter does significantly better in predicting important turning points of inflation.

There are several interesting special cases that arise from other choices of the weights. The case MV(0,1,0,1600) amounts to inverting the Phillips curve subject to the smoothness constraint. Were we to choose MV(0,0,1,1600), we would be inverting the Okun's Law relation to find potential output, again subject to the smoothness constraint. This special

29. It has been suggested to us that we extend the filter to incorporate more than one measure of inflation. In this way it might be possible to reduce the noise component associated with choosing a particular measure. It would also be possible to use the disaggregation through the production function (to apply the filter to TFP) or the link between the real wage and productivity or indeed any relationship that might offer insight into the identification of trend and cycle components of output.

case produces estimates that are similar to those produced for the United States by Clark (1983).[30] Figure 5 (p. 32) compares the base case, MV(1,1,1,1600), with the case where the Okun's Law influence is omitted: MV(1,1,0,1600). Since the estimates of the NAIRU were derived from a very similar procedure -- a bivariate version of the filter with the same link between unemployment gaps and inflation -- Okun's Law does not add very much to the information contained in the inflation relationship and the smoothness criterion.[31]

By choosing MV(0,1,0,0) or MV(0,0,1,0), we generate direct inversion of the inflation equation or the unemployment equation to obtain measures of potential output. The direct-inversion approach has been used in the past to obtain benchmark levels of the natural rate of unemployment. Normally, direct inversion leads to implausibly high and variable measures of the natural rate. There are two important reasons for retaining the curvature restriction of the filter. First, as mentioned above, for common decompositions of shocks, the H-P filter is a member of the class of optimal linear inverse filters (King and Rebelo, 1989). Provided that there are both demand and supply shocks at work, the same arguments carry over to the multivariate case. Second, inflation and unemployment contain transitory shocks that are unrelated to excess demand shocks. In this case, imposing the curvature restriction will work to minimize the spurious effects of these random disturbances on the estimates of potential output.

Figure 6 (p. 33) illustrates the differences between estimates of potential output obtained using the multivariate filter on output directly and using the same filter on TFP within the framework of a Cobb-Douglas production function, where unemployment has been filtered using the same methodology. The results are very similar.

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30. Clark developed a maximum-likelihood estimator that embodied an Okun's Law relationship and the view that productivity also contains both a temporary and a unit-root component. Since Clark did not optimize over the parameters in the Okun relationship, our method tends to produce similar estimates. The difference is that Clark employed a Kalman filter technique to build up estimates of the unit-root component.

31. This could be due, of course, to our RDXF measures of the natural rate of unemployment that determine the unemployment gap. However, alternative estimates do not appear to offer much in the way of palatable alternatives. See, for example, Rose (1988) and Coe (1990).

## 4 SOME MONTE CARLO RESULTS

Most of the papers that discuss the properties of signal extraction techniques like the H-P filter have tended to focus on the properties in large samples.[32] We are interested in the small-sample properties of these methods, in order to deal with the conditions faced in applied work. We are especially interested in the reliability of the estimates at the end of samples. These are the estimates that can most influence policy decisions. One way to address these questions is to use Monte Carlo methods to develop the relevant small-sample properties of the various techniques.

We begin, following King and Rebelo (1989), by writing down a simple, univariate decomposition of output that is quite general. This is often called an unobserved components decomposition. We assume that output is decomposable into a cyclical (or "gap") component ($Y^C$), modelled as an invertible, stationary ARMA process, and a growth (or "trend") component ($Y^P$), modelled as an integrated ARIMA process with drift.[33] The general form of the model is:

$$Y_t = Y_t^P + Y_t^C \qquad E(\nu_{P,t}, \nu_{C,t}) = 0, \forall t$$

$$D(L)(1-L)Y_t^P = \mu + F(L)\nu_{P,t} \qquad \nu_{P,t} \sim N(0, \sigma_P^2) \tag{7}$$

$$G(L)Y_t^C = H(L)\nu_{C,t} \qquad \nu_{C,t} \sim N(0, \sigma_C^2) \tag{8}$$

This model states that the log of output is driven by a combination of temporary shocks, which can have significant persistence through the propagation structure specified in equation (8), and shocks that affect output permanently, as do the $\nu_{P,t}$ in equation (7). The shocks in equation (7) have permanent effects because of the unit root, indicated by the $(1-L)$ on the left-hand side. There is also drift in the log level of $Y_t^P$, indicated by the

32. For examples, see Wiener (1949), Whittle (1963) and King and Rebelo (1989).

33. We should note that this model is quite general in the sense that it can be considered as a true reduced-form for any linear dynamic stochastic system of arbitrary order. We might also note that our qualitative results are unchanged if we model the supply shocks as being trend stationary with persistence rather than difference stationary. The important conditions are that the "supply" and "demand" shocks not be contemporaneously identifiable, and that there be some persistence of both demand and supply shocks.

parameter $\mu$. We can think of the $\nu_{P,t}$ as shocks to potential output or supply shocks. The shocks with temporary effects do not change potential output. They contribute to output gaps and are usually thought of as demand shocks.

The supposed data-generating process for output is univariate. Watson (1986), extending work by Whittle (1963), defines the optimal two-sided signal extraction filter for this decomposition. King and Rebelo (1989) develop the restrictions required for the H-P filter to satisfy the Whittle-Watson conditions.[34] Nevertheless, optimality within a class of univariate measures does not mean that efficiency gains cannot be achieved when the information set is extended. If the components of output have different effects on other observable variables, then there is every likelihood that estimates of the permanent component of output can be improved by exploiting the identifying information in the movements of those other variables. This idea parallels the standard result from econometrics that there can be efficiency gains in parameter estimation from identifying information in systems of equations.

For estimates of potential output, there is every reason to expect efficiency gains of this sort. Macroeconomic theory suggests that when there is excess demand, inflation will tend to rise. It is also widely accepted that there is a relationship between output cycles and employment cycles. In order to investigate these issues, we extend the model to include a Phillips curve and an Okun's Law relation. In addition, we allow for temporary shocks to the natural rate of unemployment. The extended model is:

$$Y_t = Y_t^P + Y_t^C \qquad E(\nu_{P,t}, \nu_{C,t}, \nu_{\pi,t}) = 0, \forall t$$

$$D(L)(1-L)(Y_t^P + \bar{U} + \alpha(\tau_{U,t} - \bar{U})) = \mu + F(L)\nu_{P,t} \tag{9}$$

$$G(L)Y_t^C = H(L)\nu_{C,t} \tag{10}$$

34. There is a correspondence between the proportion of shocks that are permanent and the value of $\lambda$. Hodrick and Prescott, as quoted by King and Rebelo (1989, p. A3) rationalize $\lambda = 1600$ for quarterly data on U.S. output as follows: "prior view that a five percent cyclical component is moderately large as is a one-eighth of one percent change in the rate of growth in a quarter. This led us to select our $\lambda^{1/2} = 5/(1/8)$ or $\lambda = 1600$ as a value for our smoothing parameter."

$$M(L)\,\pi_t = N(L)\,Y_t^C + \nu_{\pi,t} \qquad \nu_{j,t} \sim N(0, \sigma_j^2) \tag{11}$$

$$P(L)\,(U_t - \tau_{U,t}) = Q(L)\,Y_t^C + \nu_{U,t} \tag{12}$$

$$R(L)\,(\tau_{U,t} - \bar{U}) = \xi_t \qquad \xi_t \sim N(0, \sigma_\xi^2) \tag{13}$$

Equation (13) describes the evolution of the natural rate of unemployment from its long-run equilibrium level, $\bar{U}$. Equation (12) is an Okun's Law relationship, which translates excess demand in the goods market to excess demand in the labour market. Equation (11) is a Phillips curve, which posits that inflation is influenced by the output gap. Equation (10) is the process generating the cycles in the output gap, unchanged from equation (8). Equation (7) becomes equation (9); the addition of $\bar{U}$ indicates that an increase in the equilibrium value of the natural rate lowers potential output one-for-one,[35] relative to the value generated by the non-stationary process in equation (7). The other term is meant to represent the effects of cyclical movements in the natural rate around its equilibrium level. It is multiplied by labour's share of output, $\alpha$, to reflect a presumption that capital is slow to adjust to such shocks. Random shocks that increase the natural rate of unemployment will reduce potential output.

Our goal is to compare the performances of the H-P and multivariate filters, under various assumptions about the relative importance of demand and supply shocks. To do this, we need benchmark estimates for the parameters of the model. These estimates were obtained from other studies. For the Phillips curve, equation (11), the parameters are taken from Cozier and Wilkinson (1990). Following Ford and Rose (1989), a simple partial adjustment model was used in equation (12) to translate excess demand in the goods market into excess demand in the labour market. For equation (13), we borrowed a simple stochastic representation from Barro (1983), where deviations of the natural rate of unemployment from its long-run value are assumed to be persistent.

35. This is the long-term result (that is, when capital has adjusted), if the production function is Cobb-Douglas. A more complete discussion might entertain some dynamics in the transition to this long-term result. In this paper we do not consider shocks to $\bar{U}$, however, so the extra complexity is unnecessary.The $\bar{U}$ term is dropped in the empirical model used later in this paper, on the assumption that it is constant and therefore drops out of equation (9) on application of the difference operator.

The econometric literature provides little guidance on how to calibrate the demand and supply components of output movements. For our base case, we assume simple, low-order representations for both components. Based on U.S. data, Christiano and Eichenbaum (1990) estimate the contribution of supply shocks (variance of the supply shock divided by the total variance) at anywhere from 20 to 80 per cent. We consider both extremes for these numbers as well as a base case of 40 per cent, which is close to the number suggested by Cogley (1990) for both the United States and Canada. Under the base-case assumptions, the model is as follows:

$$Y_t^P + 0.65\tau_{U,t} = Y_{t-1}^P + 0.65\tau_{U,t-1} + 1.0949 + \nu_{P,t} \qquad \nu_{P,t} \sim N(0, 0.4)$$

$$Y_t^C = 1.21599 Y_{t-1}^C - 0.31306 Y_{t-2}^C + \nu_{C,t} \qquad \nu_{C,t} \sim N(0, 0.52)$$

$$\pi_t = 0.42\pi_{t-1} + 0.27\pi_{t-2} + 0.17\pi_{t-3} + 0.14\pi_{t-4} + 0.32 Y_{t-1}^C + \nu_{\pi,t}$$
$$\nu_{\pi,t} \sim N(0, 0.123)$$

$$u_t - \tau_{U,t} = 0.90(u_{t-1} - \tau_{U,t-1}) - 0.1 Y_{t-1}^C + \nu_{U,t} \qquad \nu_{U,t} \sim N(0, 0.16\times 10^{-2})$$

$$(\tau_{U,t} - \bar{U}) = 0.90(\tau_{U,t-1} - \bar{U}) + \xi_t \qquad \xi_t \sim N(0, 0.027)$$

The other two cases, with 20 and 80 per cent supply-shock ratios, use the same assumptions -- except, of course, for $\sigma_C^2$ and $\sigma_P^2$. For the most part, the precise calibration of the model is unimportant. We conducted experiments using a variety of models and obtained the same qualitative conclusions. The important assumption that drives our results is that the demand component of output is autocorrelated.

Five hundred replications under each of the three cases were computed. The results, using the multivariate filter with weighting factors MV($\eta,\theta,\gamma,\lambda$) = MV(1,1,1,1600) are shown in Figure 7 (p. 34). There are three important points to be taken from Figure 7. The first and most obvious is that the multivariate filter improves on the estimates of the H-P filter, regardless of the proportion of output variation stemming from supply disturbances. In the base case, with 40 per cent supply shocks, the 95 per cent confidence bands for the H-P filter estimates are over 6 percentage points (that is, + or - a little over 3 percentage points) for most of the sample. The multivariate filter cuts the confidence interval roughly in half.

Second, over the three cases studied, the extent of the improvement varies negatively with the proportion of shocks coming from supply sources. The H-P filter does best when supply shocks are prevalent, which suggests that the $\lambda = 1600$ assumption embodies a real-business-cycle view of the world.[36] Since the multivariate filter uses information on inflation, it improves matters most when supply shocks are less dominant.

Third, even though the multivariate filter performs better than the H-P filter, there is still considerable uncertainty in the estimates. In the historical estimates, the excess demand gap rarely exceeds 3 per cent in absolute value. Therefore, even the narrower confidence intervals from the multivariate filter imply substantial uncertainty in the point estimates.

36. Quantitatively different conclusions are obtained if $\lambda$ is chosen optimally to reflect the proportion of supply shocks; but the qualitative results remain the same so long as there are indeed demand shocks. Since one cannot know a priori what proportion of shocks are supply-side generated, it is pointless to think of varying $\lambda$ in practice.

# 5 CONCLUDING REMARKS

We have provided a brief, critical review of the kind of estimates of potential output that have been commonly used in large, econometric models and have found them wanting. In our view, insufficient attention has been paid to the reasons for estimating potential output, too much effort has been devoted to finding ad hoc variables to "fix up" equations that have broken down, and not enough credence has been given to the view that the supply side of the economy is inherently stochastic. More generally, we would argue that economists have insufficient knowledge of the structural determinants of the supply of output to recommend exploitation of econometric tools with available models. Without such a quasi-structural approach, there is a tendency to cling too long to obsolete estimates of potential output and to over-attribute apparent changes in potential output (or the natural rate of unemployment) to recognizable shocks on the basis of flimsy evidence. The fact that the quality of available data is often poor only buttresses this view.[37]

To fill an apparent need for timely and accurate estimates of potential output, we have developed a multivariate filter that uses corroborating information from ancillary variables to help identify potential output. By using the information from other variables, the multivariate filter stands on a middle ground, as a semi-structural compromise between atheoretical detrending and full, structural estimation of potential output. From this vantage, the multivariate filter provides relatively accurate estimates, very quickly and easily, while avoiding the false confidence often placed on structural estimates.

Our Monte Carlo results have pointed to another important conclusion -- that, regardless of the method used, estimates of potential output should be interpreted with caution, since the confidence bands around such estimates are quite wide. This is especially important when the estimates are used to guide macroeconomic policy decisions.

37. None of this means that structural models designed to extract potential output are useless. Where one can make progress in understanding structure, this will always be preferred to filtering techniques. Moreover, information from structural models is important in providing general ideas about information that might be useful to a filter and in determining an appropriate weighting.

We finish with a few remarks on possible extensions. There is no reason why the multivariate filter need be restricted to the variables considered in this paper. For example, one could use several inflation measures, to control for relative-price changes that may have relatively important consequences for one given measure. For that matter, the filter's use is not limited to the problem discussed here. One could use it to calibrate growth models to theoretical requirements. For example, Harrod-neutral technical change requires that the steady-state capital-output ratio be constant whenever the steady-state cost of capital is constant. But the steady-state cost of capital is not directly observable. The multivariate filter could be used to give an idea of how much movement in the capital-output ratio can be explained by movements in the steady-state cost of capital, and how much is explained by "disequilibrium."

**Figure 1: Potential Output Using Trend-Through-Peaks Method**

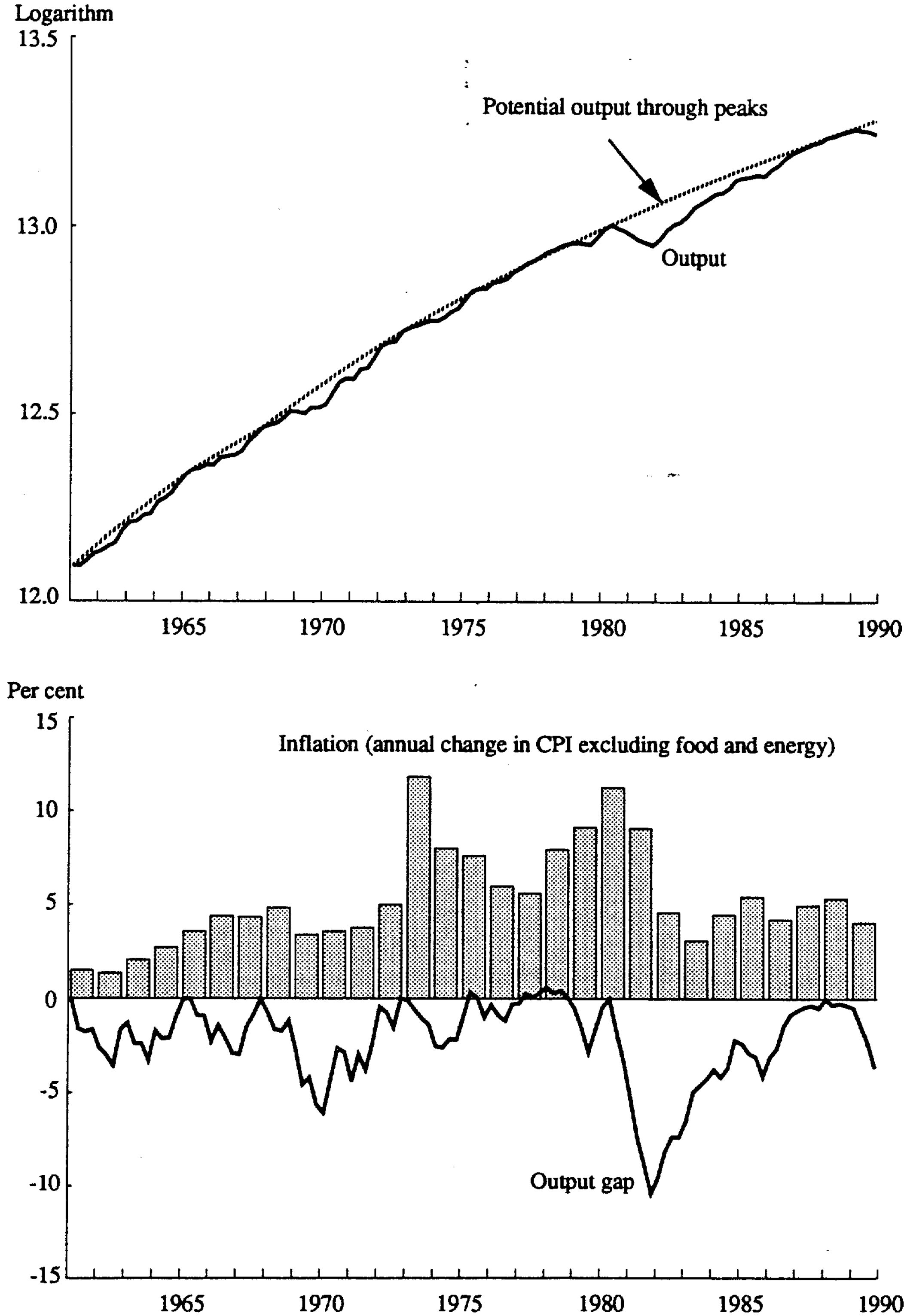


**Figure 2: Estimate of Commercial Sector Potential Output**

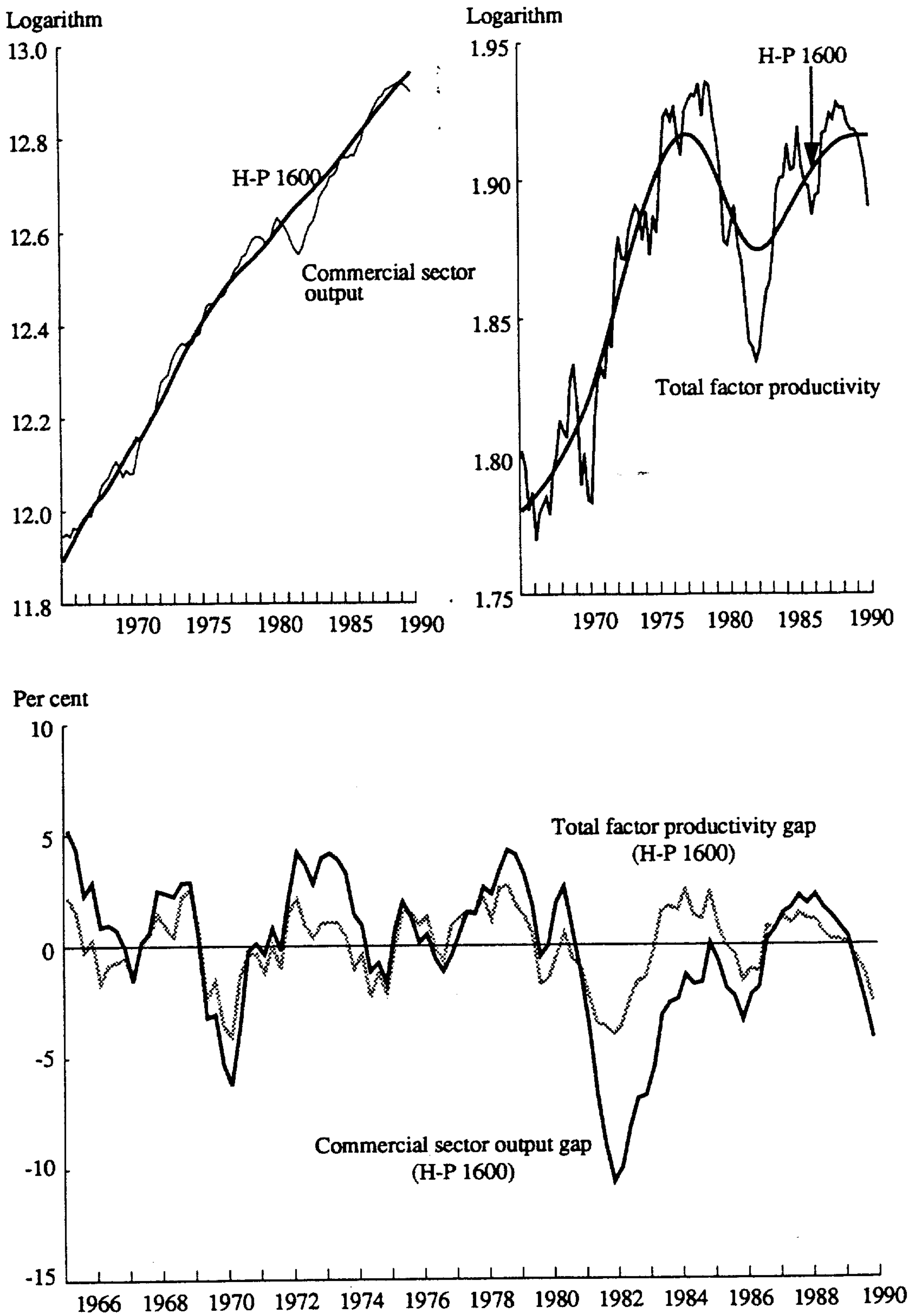
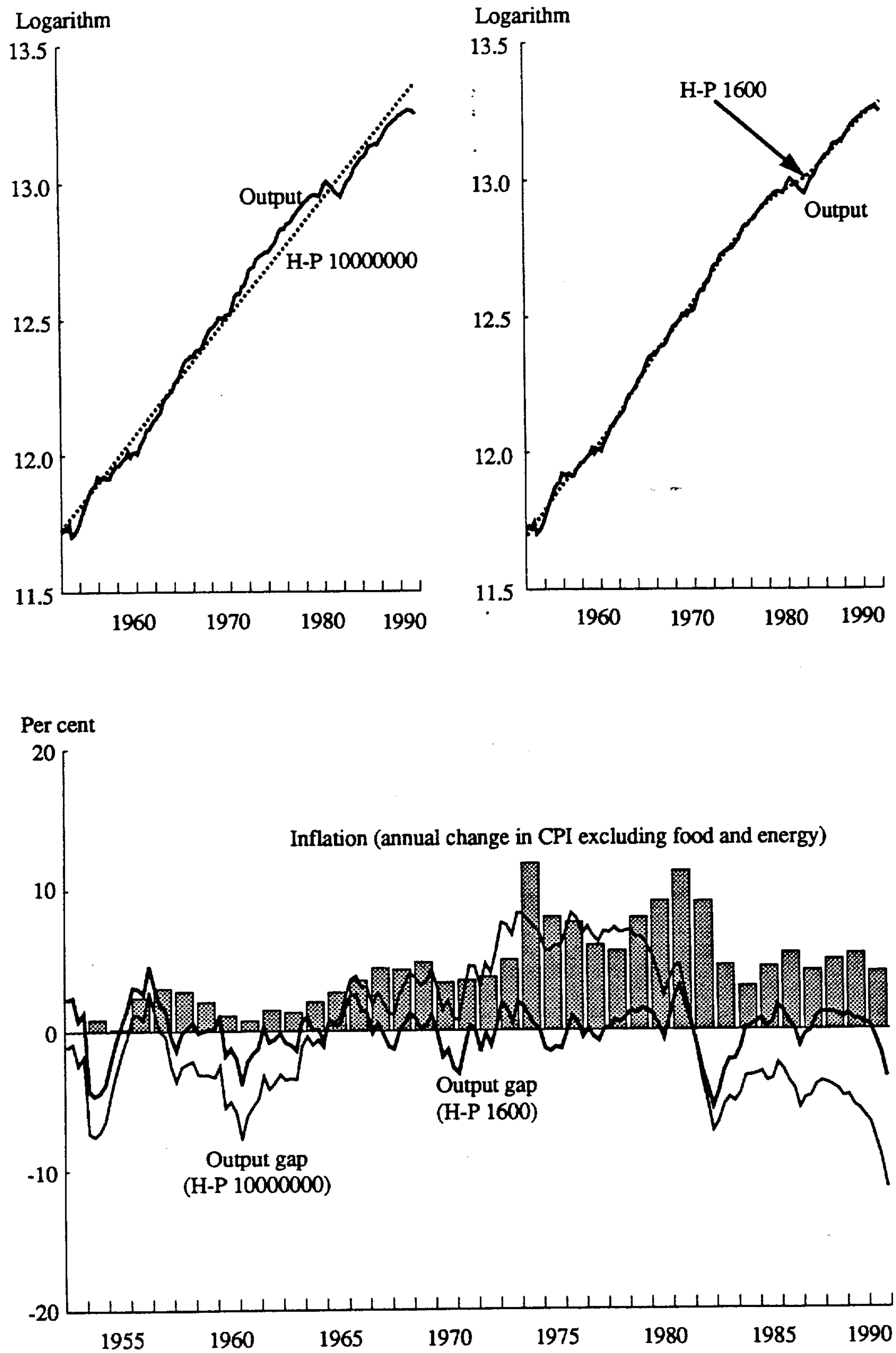
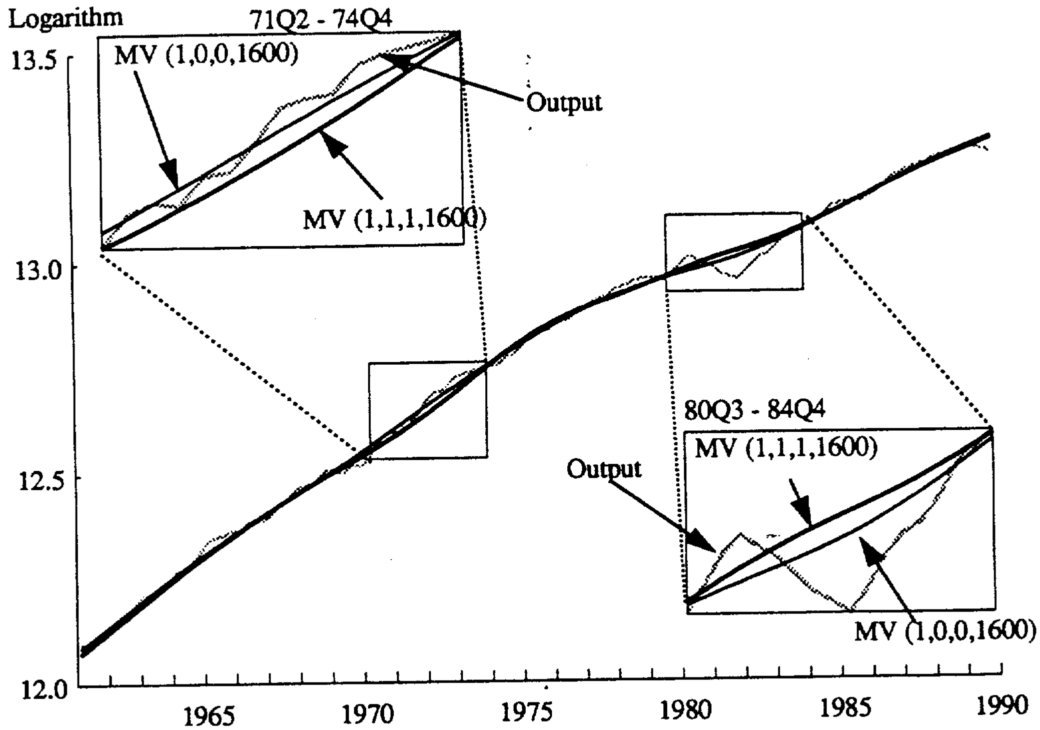
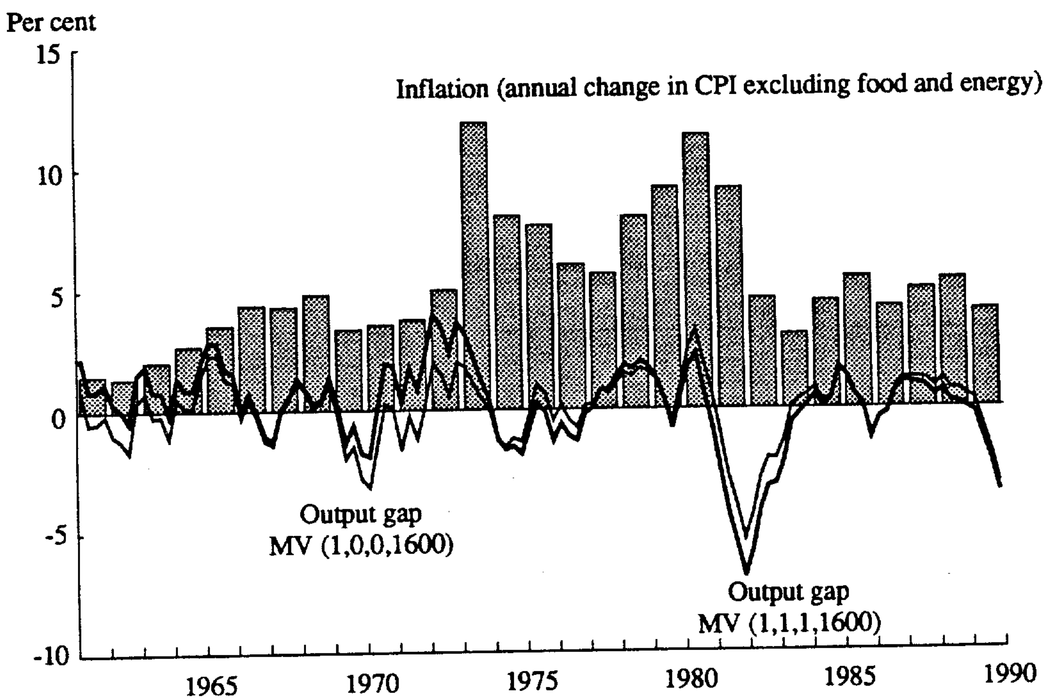


**Figure 3: Measures of Potential Output from the H-P Filter**



## Figure 4: Estimates of Potential Output Using the Multivariate Filter: {MV(1,1,1,1600) and MV(1,0,0,1600)}



## Figure 5: Estimates of Potential Output Using the Multivariate Filter: {MV(1,1,1,1600) and MV(1,1,0,1600)}

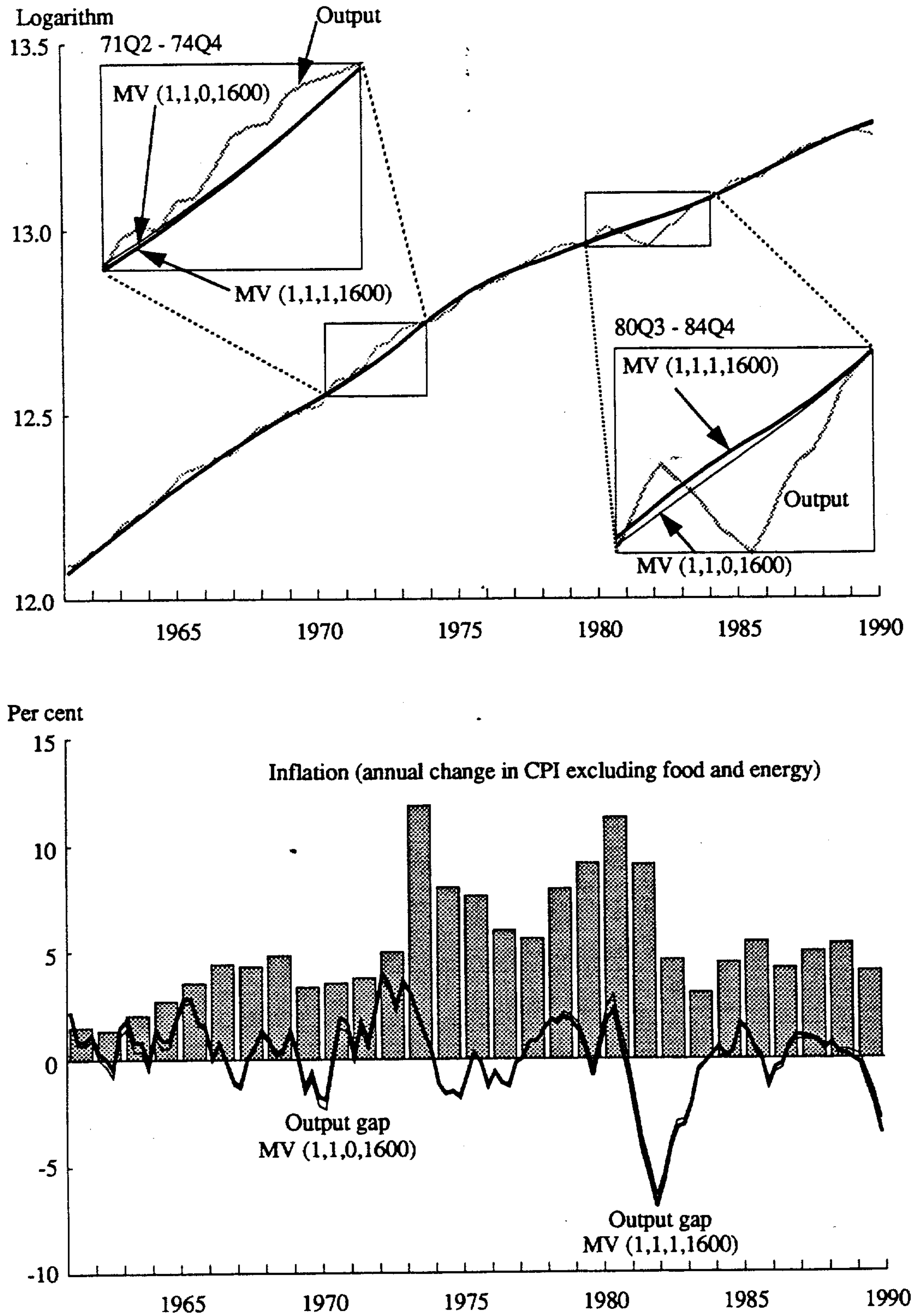
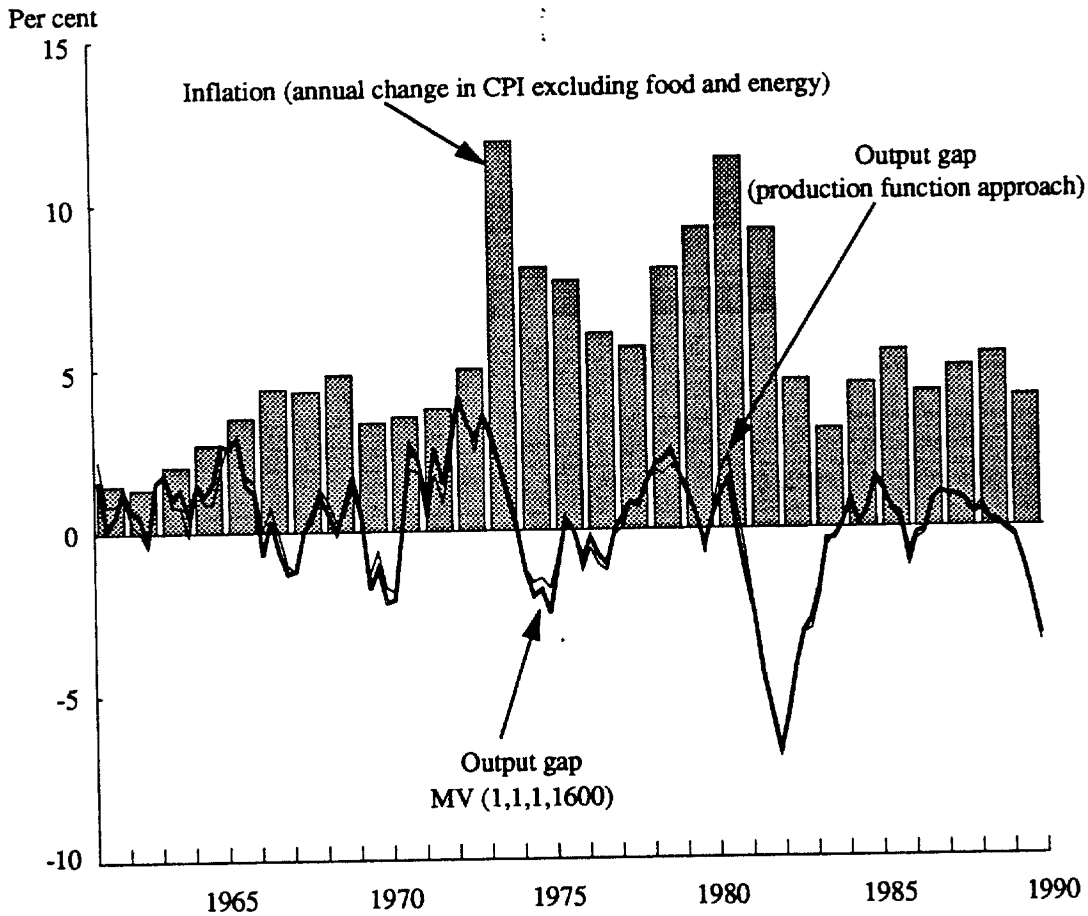
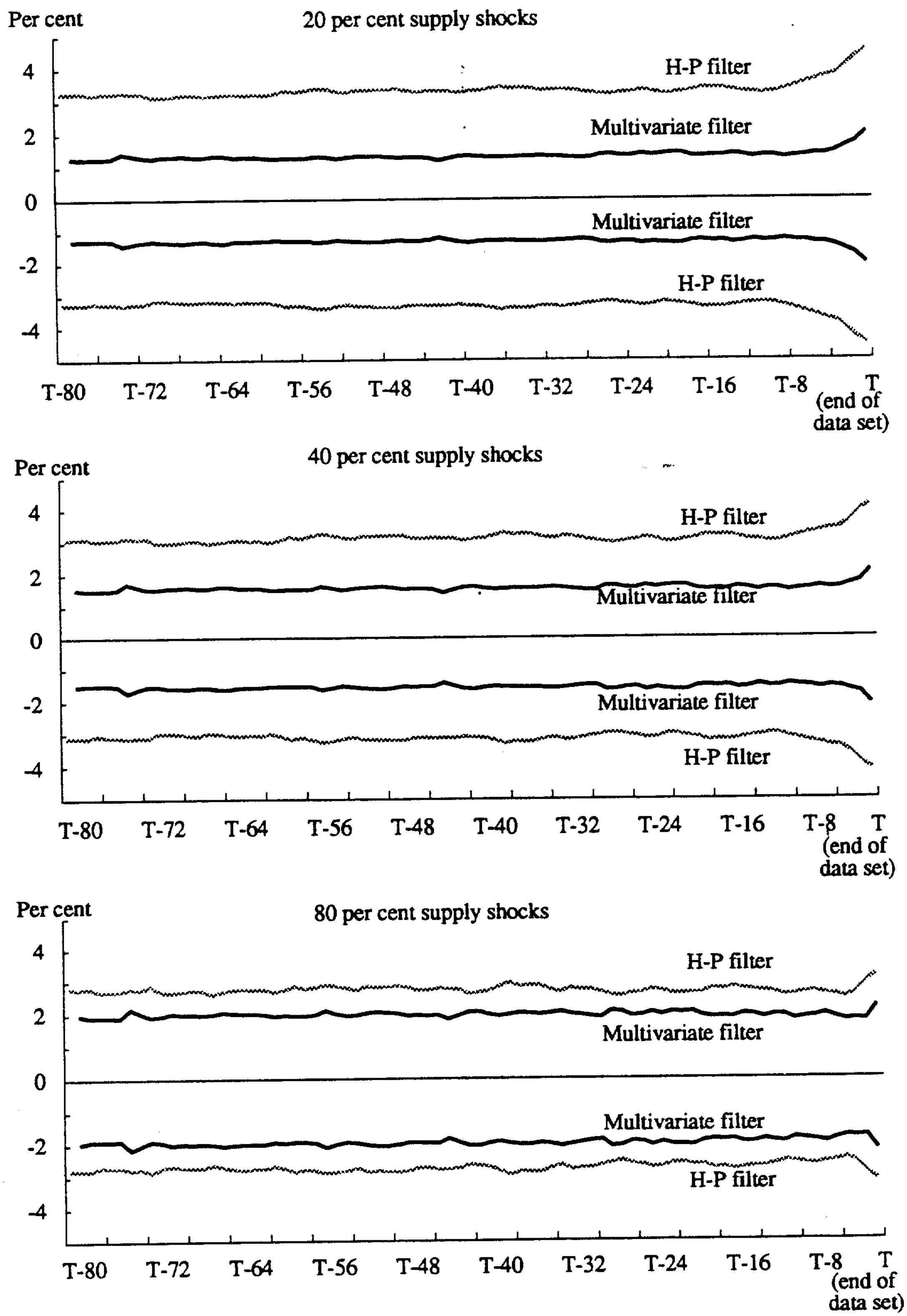


**Figure 6: Estimates of the Output Gap from a Production Function With Trend Total Factor Productivity and from MV(1,1,1,1600)**



## Figure 7: 95 Per Cent Confidence Bands for Estimates of the Output Gap (Based on the Monte Carlo Experiments)



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