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Some Further Consequences of Mismeasuring Potential Output

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ABSTRACT

In this paper, we report results of a Monte Carlo study designed to examine the power of direct tests for non-linearity. The example we use is the Phillips curve. We specify a small model of output and inflation, which includes an important non-linearity in the Phillips curve. The model is calibrated to reflect the properties of the Canadian data. It is then used to generate a number of hypothetical sequences of observations on output and inflation. Output is generated by an unobserved components model — the econometrician observes output but cannot observe potential output or the output gap that influences inflation. Using a number of standard, univariate techniques to measure potential output, we simulate what the econometrician would find in estimating the Phillips curve and testing for non-linearity. We give the econometrician the best possible chance of discovering the non-linearity by assuming that the precisely correct functional form is estimated, including the dynamics arising from expectations. However, our results indicate that the econometrician is quite likely to reject the existence of the non-linearity. The fact that the key determinants of inflation are unobservable makes the estimates imprecise and compromises statistical inference. We also show that this problem is reduced, but not eliminated, if potential output is measured by means of a multivariate filtering procedure that uses information about inflation in identifying the output gap.

RÉSUMÉ

Dans cette étude, les auteurs présentent les résultats des simulations de Monte-Carlo qu'ils ont effectuées afin d'analyser, en prenant comme exemple la courbe de Phillips, la puissance des tests directs de non-linéarité. Ils spécifient un petit modèle de production et d'inflation qui intègre une courbe de Phillips affichant une forte non-linéarité. Après l'avoir adapté de manière à ce qu'il reflète les propriétés des données canadiennes, les auteurs se servent du modèle en question pour produire plusieurs séries d'observations hypothétiques sur la production et l'inflation. Les données de la production sont générées à l'aide d'un modèle à composantes non observées (les économétriciens peuvent observer la production, mais ne peuvent observer ni la production potentielle ni le déséquilibre de la production qui influence l'inflation). En se servant de diverses techniques traditionnelles, univariées, pour mesurer la production potentielle, les auteurs simulent les résultats qu'un économétricien obtiendrait s'il procédait à une estimation de la courbe de Phillips et à un test de non-linéarité. Ils font l'hypothèse que la forme fonctionnelle qui est estimée, y compris la dynamique générée par les anticipations, est connue de façon précise, afin de donner à l'économétricien les meilleures chances possibles de découvrir la non-linéarité. Les résultats obtenus montrent toutefois qu'il est fort probable que l'économétricien rejette la non-linéarité. Le fait que les principaux déterminants de l'inflation ne sont pas observables rend les estimations imprécises et compromet la validité des inférences statistiques. Par ailleurs, les auteurs montrent qu'on atténue le problème, sans le supprimer, en mesurant la production potentielle à l'aide d'une technique de filtrage à plusieurs variables qui met à contribution des renseignements sur l'inflation dans le calcul du déséquilibre de la production.

1. Introduction

In a recent paper, Laxton, Shoom and Tetlow (1992) showed, based on a Monte Carlo experiment, that persistent errors in the measurement of potential output can lead researchers to false conclusions concerning the Phillips curve. In particular, they showed that the coefficient on the change in the output gap is likely to be statistically significant in estimated linear Phillips curves, even though, by construction in the Monte Carlo experiment, no such effect is there. The same research showed that the econometrician is likely to underestimate the true coefficient on the output gap itself.

Using similar methodology, we show here that researchers may have difficulty finding statistically significant, non-linear structure in Phillips curves, where such non-linearity is a feature of the true data-generating process. The reason for this is essentially the same as in the earlier study — errors in measuring the state of excess demand make it very difficult to arrive at correct statistical inferences regarding the properties of a model intended to represent the effects of excess demand on other economic variables. This point is of great importance in forecasting and monetary policy analysis, because the relationships that are central to the nominal dynamics of an economy are subject to this kind of uncertainty.

It has been a common argument that excess demand has a larger effect on inflation than does excess supply.[1] This idea was an important part of the original Phillips curve, and most of the early work on wage Phillips curves incorporated an important non-linearity of some form. Yet, in the literature one finds at best only weak statistical support for a non-linearity in price Phillips curves.[2] We think that this can be explained, in part, by the methodology used to measure potential output and therefore the output gap.

Most techniques used to measure potential output employ a mean-squared-error (MSE) criterion to define a curve representing potential output as a measure of central tendency of the series for actual output. If the Phillips curve is non-linear, however, such that excess demand tends to be more inflationary than excess sup-

1. It should be understood that this is the kind of non-linearity we have in mind throughout this paper. In the next section we make the statement in the text more precise by specifying a non-linear model.
2. For example, recent work done at the Bank of Canada by Cozier and Wilkinson (1990, 1991), reports tests where a restriction to linearity is not rejected statistically. Chadha, Masson and Meredith (1992) reject linearity in a formal statistical test in a pooled sample of G-7 countries, but discard the results as not strong enough to be convincing.

ply is deflationary, then symmetrically distributed demand shocks will have an inflationary bias and a monetary authority pursuing an inflation target will be forced to counteract that bias. We have shown elsewhere that such a world will be characterized by output distributions skewed to the left (a tendency to larger recessions).[3] The MSE criterion would weight the large negative numbers too heavily and result in too many and too large estimates of excess demand.[4] We have also shown elsewhere that standard filtering methods for measuring potential output are likely to be quite imprecise.[5]

Suppose the economy is non-linear in the sense described above. What will be the implications for estimated Phillips curves? From an econometric point of view, there are two difficulties to consider. First, the coefficient(s) on the excess demand components will be biased downwards and the coefficient(s) on the excess supply component(s) will be biased upwards. Second, even in the best of circumstances, where the econometrician is fortunate enough to get, by chance, accurate estimates of the gaps, the small-sample properties of OLS estimators will be poor. This will be the case, in part because there will tend to be brief stints of excess demand followed by sharper excess supplies or long "recovery" periods; typical samples used by econometricians will not contain enough cycles to give a great deal of identifying power to the estimator.[6]

In this paper, we show that these econometric difficulties also lead to problems in identifying a non-linearity that is truly there, by construction. Given that the positive gap measures from MSE methods tend to be too large, the estimator is forced to lower the supposed effect of excess demand on inflation. It would be surprising if, at the same time, tests for non-linear effects would not be compromised. In this paper, using Monte Carlo simulations, we demonstrate that such tests will indeed be substantially biased against finding the non-linearity.

3. See Laxton, Rose and Tetlow (1993b).
4. This will be true of sophisticated modern detrending techniques, such as the Hodrick-Prescott (H-P) (1980) filter. The H-P filter derives trend values for a series by minimizing the MSE, subject to a curvature restriction. The curvature restriction may have an important effect on the estimates, but it does not eliminate the problem identified here.
5. See Laxton and Tetlow (1992).
6. The estimation bias problem described in this paper is similar to that of Laxton, Shoom and Tetlow (LST) (1992) in that mismeasurement of potential output is the root of the problem, but it differs in the nature of mismeasurement. LST considered shocks to potential that produced autocorrelated errors in the measurement of the output gaps. Since they assumed a linear Phillips curve, however, there was no asymptotic bias in the estimated gaps. Here we note the possibility that the average size of output gaps could be incorrect owing to the incorrect assumption embedded in their construction that positive gaps have the same absolute effect on inflation as negative gaps.

From a policy perspective, two types of problems arise if the Phillips curve is non-linear. First, actions based on the presumption that the Phillips curve is linear will result in systematic errors in achieving targets; inflation will tend to be, on average, higher than desired. Second, if interest rates do not adjust to moderate quickly any excess demand, inflation will get entrenched in expectations and a much larger or prolonged policy reaction will be required later. Forecasters will have the same kinds of problems. There will be a tendency to underpredict inflation when it is rising in response to excess demand, and to be surprised at how fast inflation can escalate. Conversely, forecasters will be frustrated by the persistence of inflation in the face of tight monetary conditions and substantial output gaps. In both cases, much judgment will have to be added to the model's predictions to keep forecasts close to reality.

The model and specification issues are discussed in section 2. The Monte Carlo results are described in section 3. In section 4, we offer some further thoughts on the specification, measurement and estimation issues involved and some concluding remarks on the implications of the results for policy debates.

2. The model

Consider an economy that, in the absence of monetary intervention, would generate a path for output from three sources: deterministic upward drift over time, a shock that raises (or lowers) actual and potential output simultaneously, and a shock that influences the gap between actual and potential output. The former shock, which is often called a supply shock, is usually thought of as permanent, as it is in Blanchard and Quah (1989) and Dea and Ng (1990), for instance, but it need not be so. All that is required is that it have high persistence. The latter shock, which is often called a demand shock, must be temporary — since, by definition in models with a stable steady state, output gaps cannot persist indefinitely — but it can have quite persistent effects. Indeed, most empirical estimates based on Canadian data do suggest substantial persistence of output gaps. This characterization of output is often called an unobserved components decomposition.[7] Following Laxton and Tetlow (1992), we use such a decomposition, calibrated to reflect the Canadian data.

In accordance with time-series evidence, equation (2) posits that potential output (Y_t^P) evolves according to a process with a unit root and drift.[8] The degree of drift is chosen to approximate historical Canadian data. The data also tell us that the cyclical (demand) portion of total output (Y_t^C) can be fairly well represented by the AR(2) process shown in equation (3).[9] Finally, suppose that the monetary authorities influence interest rates in an attempt to target inflation (the reaction function is described below). The actions of policy have an effect on the state of excess demand, as shown in equation (4). RR should be thought of as the real interest rate, relative to some control value that would be generated in the absence of monetary intervention. We postulate that policy has effects that cumulate gradually over eight quarters. Thus equation (3) represents the cyclical properties of the uncontrolled economy and equation (4) overlays monetary control effects, working through the real interest rate, on top of that "natural" cyclical structure. Econometricians observe output, but cannot observe directly whether there has been a supply shock or a demand shock or some combination of the two. In other words,

7. See Blanchard and Fischer (1989, 5-12) for a discussion of the decomposition of output into trends and cycles.
8. See Macklem (1991), where this formulation is tested and not rejected, using Canadian data.
9. This simple autoregressive propagation mechanism can be thought of as a reduced-form approximation to a general, linear, dynamic, stochastic model of the economy. Higher-order representations might be necessary to exhaust the exploitable correlations in the data. However, since the results of our experiments will depend only on the dominant roots of the process, the second-order representation is adequate for our needs.

they cannot observe the true output gap. Econometricians are also presumed ignorant of the precise impact of monetary interventions on output, at least to the extent of not being able to use such information to help identify $(YGAP_t)$. Moreover, the problem is not limited to levels. Since not all fluctuations in output imply fluctuations in the output gap, econometricians cannot know how the gap has changed from one quarter to the next. To estimate the Phillips curve, a proxy for potential output and the output gap must be provided.

Output (Y), potential output (YP) and the output gap (YGAP):

$$Y_t = YP_t + YGAP_t \tag{1}$$

$$YP_t = YP_{t-1} + 0.010949 + \varepsilon_{P,t} \tag{2}$$

$$YC_t = 1.21599YC_{t-1} - 0.31306YC_{t-2} + \varepsilon_{C,t} \tag{3}$$

$$YGAP_t = YC_t - 0.8\sum_{j=1}^{8} RR_{t-j}/8 \tag{4}$$

Inflation (Π) and inflation expectations (Π^e):

$$\Pi_t = \Pi_t^e + 0.32\sum_{j=1}^{8} YGAP_{t-j}/8 + 0.19\sum_{j=1}^{4} POSGAP^2{}_{t-j}/4 + \varepsilon_{\Pi,t} \tag{5}$$

$$POSGAP = \begin{pmatrix} YGAP\ldots \text{ if YGAP>0} \\ 0 \ldots \text{ otherwise} \end{pmatrix} \tag{6}$$

$$\Pi_t^e = 0.4237\Pi_{t-1} + 0.2744\Pi_{t-2} + 0.1673\Pi_{t-3} + 0.1346\Pi_{t-4} \tag{7}$$

Inflation target path (ΠTAR) and the real interest rate (RR):

$$\Pi TAR_t = 0.9\Pi_{t-1} + 0.1\Pi SS \tag{8}$$

$$RR_t = 2.0\,[\Pi_t - \Pi TAR_t] + YGAP_t \tag{9}$$

Inflation is presumed to be generated according to the non-linear Phillips curve shown in equation (5). Inflation responds symmetrically to a lagged eight-quarter moving average of the output gap. The non-linearity comes from a four-quarter moving average of the squared values of the positive gaps; that is, the entry in the four-quarter average is zero if the measured gap is not positive. This means that excess demand creates more inflationary pressure than excess supply creates deflationary pressure. Inflation is also influenced by expectations of inflation, which are described by equation (7).[10] This particular specification comes from Cozier and Wilkinson (1990), where expectations were assumed to be entirely backward-looking, with a unit sum imposed on the coefficients, in common with much recent empirical work on the subject. Finally, there is an independent disturbance or "shock," $\varepsilon_{\Pi, t}$, applied to inflation.

The monetary authority is presumed to have a long-term target for inflation, ΠSS. However, because of lags and inertia in the economy, the monetary authority does not try to bring the economy immediately to the long-term target. Rather, policy action at any point in time is based on a shorter-term target, defined as a weighted average of the long-term target and the most recent outcome. The reaction function is shown in equation (9). The authorities raise interest rates if inflation is above the short-term target and/or if there is a positive output gap. We show a low weighting on the long-term target in equation (8). We investigate the sensitivity of the results to this assumption by repeating the experiments with an equal weight on the two terms in equation (8).

The data cannot provide direct measures of the variances of $\varepsilon_{C, t}$ and $\varepsilon_{P, t}$ since the two are not independently observable. For these values we turn to the literature on the stochastic properties of output. Christiano and Eichenbaum (1990) estimate the proportion of total output variation in the U.S. data that is attributable to permanent shocks, $\sigma^2_{\Delta YP}/(\sigma^2_{\Delta YP}+\sigma^2_{\Delta YC})$, at anywhere between 20 and 80 per cent. Cogley (1990) places his best guess for both Canada and the United States at about 40 per cent. Dea and Ng (1990) find a value of 80 per cent for Canada. We report Monte Carlo tests using all three of these figures. For our base case, we assume the intermediate value of 40 per cent. This implies the following standard deviations for the disturbance terms in equations (2) and (3): $\{\sigma_P, \sigma_C\} = \{0.63245, 0.7219\}$, measured as percentages.[11] One hundred draws were taken from these distributions to generate hypothetical quarterly data for output for a sample of 22 years' duration (that is, 88 quarterly observations).[12] Corresponding output gap data

10. The fact that expectations are also unobservable poses another serious difficulty for the econometrician. For this paper, however, we focus on the problems associated with the mismeasurement of the output gap and suppress most of the problems associated with measuring inflation expectations.

11. The derivation of these results is described in the Appendix.

12. The actual data generation began well before the first point called "data" here. The sample is the one presumed used by the econometrician.

were generated using equation (4) and the identities. The inflation data come from equation (5) and the associated identities. The disturbances for equation (5) were generated from a distribution with standard deviation 1.4, consistent with the results reported in Cozier and Wilkinson (1990, 1991).

We test the sensitivity of the results to the relative noise variances by repeating the experiments using a standard deviation of 0.7 for the disturbances in equation (5) as well as by altering the relative variability of the permanent and transitory shocks to output.

The econometric problem comes in inferring the output gap $(YGAP_t)$ when only aggregate output (Y_t) is observable. We consider several possible approaches to this problem. For example, in accordance with current common practice, we measure trend output using the Hodrick-Prescott (H-P) (1980) filter. With these trend values as estimates of potential output, we compute the implied output gaps, just as one would using the H-P filter on real-world data. We investigate the sensitivity of the results to the choice of filtering technique in two ways. We try a standard quadratic trend measure and a measure based on the multivariate filter proposed by Laxton and Tetlow (1992). This alternative filter extends the H-P filter by using information about inflation in identifying the output gap.[13]

We then consider what an econometrician would find in estimating the Phillips curve. We give the econometrician a great deal of information. The functional form is assumed to be known precisely, including the formulation of inflation expectations. This is an unrealistic assumption that greatly simplifies the econometric problem. We have chosen to do this to keep the inference question as clear as possible and directly focussed on the uncertainty concerning potential output. We find that, despite this quite accurate empirical specification, the econometrician is not likely to identify the true data-generating process for inflation from regressions using estimated output gaps.

13. The extra information comes from a linear Phillips curve of the type reported by Cozier and Wilkinson (1990, 1991). We do not exploit the information about the non-linearity in generating the alternative output gap measures. The measures are constructed using a linear system of equations similar to the one in Ford and Rose (1989).

3. Results

The base-case results are shown in Table 1. "Base case" refers to the intermediate variance ratio (40 per cent supply shocks) and the parameters shown in equations (1) to (8) above.[14] The test for non-linearity is a classical t-test on the coefficient on the POSGAP variable in the estimated version of equation (5). A false rejection means a rejection at the 5 per cent significance level (or the 95 per cent confidence level), when the hypothesis is in fact true.

Table 1 reports eight experimental structures. The first two are the results when the H-P smoothing parameter is set to 1600 — the value chosen by Hodrick and Prescott themselves and by many others since. These two cases compare the results for two sets of weights in the setting of the short-term target for inflation by the monetary authority. Experiment 1 is a case where the authority does not pursue its target very aggressively and allows a root close to unity in the inflation process. The short-term target is set with a weight of 0.9 on the recent value and just 0.1 on the long-term target. This case is relatively consistent with the assumed expectations specification. In experiment 2, the monetary authority is much more

Table 1: Results of the Monte Carlo experiment: base-case model

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	86/100	0.361
2	HP1600	0.5	0.5	70/100	0.272
3	HP2500	0.9	0.1	86/100	0.366
4	HP2500	0.5	0.5	72/100	0.273
5	Quadratic	0.9	0.1	90/100	0.467
6	Quadratic	0.5	0.5	77/100	0.339
7	MV1600	0.9	0.1	30/100	0.088
8	MV1600	0.5	0.5	11/100	0.036

14. Except, of course, we vary the weight on the long-run inflation target in equation (8).

aggressive and gives equal weight to last period's inflation and to the long-term target level of inflation in setting the short-term target. In experiment 2, the non-linearity is falsely rejected in 70 per cent of the trials. When monetary policy is less aggressive, so that shocks to inflation tend to persist longer, the rate of false rejection rises to 86 per cent. The final column shows the average significance level for this test across the 100 trials. Clearly, the false rejections are not marginal rejections, on average.

Experiments 3 and 4 repeat this exercise with a larger value for the H-P smoothness parameter (moving the proxy for potential output towards a linear trend, but remaining far from the high value necessary to make the result look like a straight line). The results are essentially the same.

Experiments 5 and 6 repeat the exercise using an estimated quadratic time trend as a proxy for potential output. The incidence of false rejection of non-linearity is somewhat higher in these experiments, 90 per cent in the close-to-unit root world of experiment 5.

Finally, experiments 7 and 8 show the results when the Laxton-Tetlow (1992) multivariate filter is used to measure potential output. The false-rejection rates are substantially lower, as are the average significance levels. While this is encouraging from the perspective of possible gains from using the multivariate approach, the false-rejection rates from the standard classical test are still at least double the proportion to be expected by chance.

We have tried a number of sensitivity tests on these conclusions. Table 2 reports results when we cut the standard deviation of the inflation shock in half, from 1.4 to 0.7, thus presumably giving the estimator a better chance of identifying the form of link between excess demand and inflation. The false-rejection rates and average significance levels are somewhat lower than those in the base case, but the qualitative conclusions of the experiments are not affected.

Table 3 shows the results with the effect of the non-linearity doubled (that is, increasing the coefficient on the POSGAP term to 0.38). Again, one would expect this to help the estimator identify that a non-linearity is there in the process generating the data. The results do show a reduction in false-rejection rates and average significance levels, relative to the base case, but again not enough to change the basic conclusions.

Table 2: Results of the Monte Carlo experiment: $\sigma_\pi = 0.7$

Experiment	Detrending technique	Policy rule parameters π_{t-1}	πss	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	65/100	0.268
2	HP1600	0.5	0.5	52/100	0.179
3	HP2500	0.9	0.1	67/100	0.268
4	HP2500	0.5	0.5	53/100	0.180
5	Quadratic	0.9	0.1	78/100	0.332
6	Quadratic	0.5	0.5	67/100	0.238
7	MV1600	0.9	0.1	19/100	0.048
8	MV1600	0.5	0.5	10/100	0.021

Table 3: Results of the Monte Carlo experiment: doubled true non-linearity

Experiment	Detrending technique	Policy rule parameters π_{t-1}	πss	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	65/100	0.262
2	HP1600	0.5	0.5	43/100	0.159
3	HP2500	0.9	0.1	66/100	0.264
4	HP2500	0.5	0.5	43/100	0.160
5	Quadratic	0.9	0.1	82/100	0.346
6	Quadratic	0.5	0.5	60/100	0.206
7	MV1600	0.9	0.1	17/100	0.052
8	MV1600	0.5	0.5	9/100	0.022

Next, Table 4 shows the results when we lower the proportion of "supply" shocks to 20 per cent from 40 per cent in the base case. Since more of the variation in observed output now truly reflects variation in the output gap, it is to be expected that the detrending techniques would produce better measures of the gap and that the estimator would therefore have better success in identifying the non-linearity. We do see this result, but once again the change is not substantial enough to change the qualitative conclusions of our experiments.

Table 4: Results of the Monte Carlo experiment: 20% supply shocks

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{SS}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	72/100	0.282
2	HP1600	0.5	0.5	54/100	0.201
3	HP2500	0.9	0.1	73/100	0.286
4	HP2500	0.5	0.5	55/100	0.205
5	Quadratic	0.5	0.5	84/100	0.321
6	Quadratic	0.5	0.5	64/100	0.208
7	MV1600	0.9	0.1	23/100	0.063
8	MV1600	0.5	0.5	9/100	0.027

We also repeated the sensitivity tests of Tables 2 and 3 with the smaller supply-shock variance of Table 4. These experiments are shown in Tables 5 and 6. The proportional reduction in the false-rejection rate is greater (for example, the changes between Tables 4 and 5 are proportionally larger than those between Tables 1 and 2), but the central point remains valid. The econometrician, at least one using classical tests and standard methods to measure potential output, is quite likely to falsely reject the hypothesis that the structure is non-linear. An exception arises in the case of the multivariate filter. In both Table 5 and Table 6, with an aggressive monetary control rule, the proportion of false rejections drops to the 5 per cent level to be expected by chance. It is noteworthy that the average significance levels are generally much lower in Tables 5 and 6. The false rejections are closer to being marginal when all the conditions are favourable to identifying the non-linearity.

Table 5: Results of the Monte Carlo experiment: 20% supply shocks, $\sigma_\pi = 0.7$

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	44/100	0.168
2	HP1600	0.5	0.5	32/100	0.103
3	HP2500	0.9	0.1	45/100	0.167
4	HP2500	0.5	0.5	28/100	0.104
5	Quadratic	0.9	0.1	50/100	0.182
6	Quadratic	0.5	0.5	33/100	0.107
7	MV1600	0.9	0.1	8/100	0.021
8	MV1600	0.5	0.5	5/100	0.010

Table 6: Results of the Monte Carlo experiment: 20% supply shocks, doubled true non-linearity

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	50/100	0.180
2	HP1600	0.5	0.5	28/100	0.095
3	HP2500	0.9	0.1	50/100	0.177
4	HP2500	0.5	0.5	29/100	0.097
5	Quadratic	0.9	0.1	57/100	0.189
6	Quadratic	0.5	0.5	34/100	0.100
7	MV1600	0.9	0.1	11/100	0.031
8	MV1600	0.5	0.5	5/100	0.014

We ran the same battery of tests assuming a higher proportion of supply shocks — the 80 per cent figure from the literature. The results are shown in Tables 7, 8 and 9. They mirror those above in that the proportion of false rejections rises everywhere. This is to be expected, since the standard filtering techniques are erring more in assigning too much output variation to the gap used in estimating the Phillips curve. This evidently lowers substantially the power of tests concerning the functional form with respect to that gap measure. In many cases the hypothesis of non-linearity is almost always falsely rejected. Moreover, in this case there is relatively little improvement in the false-rejection rate as we lower the variance of the inflation shock or increase the extent of non-linearity. The same conclusion is apparent from the average significance levels, which are notably higher in these tables. Except in the cases where the multivariate filter is used, the econometric results give virtually no signal that there may be a non-linearity in the Phillips curve. The results for the multivariate filter are encouraging. The false-rejection rates are still very high, but the significance levels do not deteriorate as they do in the other cases. This suggests that unless the econometrician sticks rigidly to the standard classical t-test, the estimation results with an MV filter will often provide an indication that a non-linearity is present.

Table 7: Results of the Monte Carlo experiment: 80% supply shocks

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	97/100	0.493
2	HP1600	0.5	0.5	91/100	0.452
3	HP2500	0.9	0.1	96/100	0.504
4	HP2500	0.5	0.5	92/100	0.457
5	Quadratic	0.5	0.5	96/100	0.514
6	Quadratic	0.5	0.5	92/100	0.481
7	MV1600	0.9	0.1	47/100	0.141
8	MV1600	0.5	0.5	27/100	0.059

Table 8: Results of the Monte Carlo experiment: 80% supply shocks, $\sigma_\pi = 0.7$

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	96/100	0.486
2	HP1600	0.5	0.5	89/100	0.451
3	HP2500	0.9	0.1	97/100	0.491
4	HP2500	0.5	0.5	89/100	0.456
5	Quadratic	0.9	0.1	92/100	0.506
6	Quadratic	0.5	0.5	94/100	0.474
7	MV1600	0.9	0.1	59/100	0.192
8	MV1600	0.5	0.5	32/100	0.104

Table 9: Results of the Monte Carlo experiment: 80% supply shocks, doubled true non-linearity

Experiment	Detrending technique	Policy rule parameters π_{t-1}	π_{ss}	Proportion of false rejections of non-linearity	Average significance level
1	HP1600	0.9	0.1	95/100	0.481
2	HP1600	0.5	0.5	81/100	0.370
3	HP2500	0.9	0.1	95/100	0.491
4	HP2500	0.5	0.5	82/100	0.375
5	Quadratic	0.9	0.1	94/100	0.519
6	Quadratic	0.5	0.5	90/100	0.436
7	MV1600	0.9	0.1	37/100	0.112
8	MV1600	0.5	0.5	17/100	0.041

4. Concluding remarks

Our conclusions can be brief. The experiments we have done are designed to show that the world may well be non-linear, even though econometric tests may suggest otherwise. The fact that the key determinants of inflation are unobservable makes the econometric problem very difficult and statistical inference imprecise. Our experiments have shown that the estimation results are likely to be biased and that econometricians are quite likely to draw false inferences in classical hypothesis tests concerning the form of the link between excess demand and inflation.

The fact that we have given the econometrician a great deal of information makes this conclusion all the more striking. Recall that the precisely correct functional form is estimated. In particular, we have suppressed all problems of identifying expected inflation. In "real-world" empirical work, the absence of direct measures of expectations adds another dimension to the problems of statistical inference in Phillips curves. We consider it most unlikely that adding a second source of specification error would reduce the problems associated with mismeasurement of potential output, but that question remains as an interesting topic for future research.

We think that the logical case for a non-linear Phillips curve is appealing. However, we do not suggest that an empirical case for a non-linear specification emerges from our Monte Carlo results. Such a case must be made through examining forecasting errors, testing for non-linearity, searching for other corroborating evidence for non-linearity and so on. This paper is designed to address one particular question — if there is a non-linearity, why is it that we often do not find strong empirical evidence for it in econometric tests? We have provided an answer to this question in the case of the Phillips curve. Econometricians may falsely reject non-linearity because of difficulties in representing the unobservable output gap. These results do not, by themselves, demonstrate that the world is truly non-linear, but we think that they demonstrate that it is dangerous to rely on standard classical test results to rule out non-linearity in a model with an unobservable explanatory variable.

In considering the possible gains from using the multivariate filtering technique for measuring potential output (Laxton and Tetlow, 1992), we found encouraging evidence that this technique does help in the identification of a non-linearity in the Phillips curve. Use of the technique does not eliminate the bias in the standard classical test, but it does change the character of the estimation results such that an econometrician will generally see an indication of the non-linearity in the results. It is especially encouraging that this gain was apparent even when we assumed a high proportion of supply shocks.

Appendix

Setting the error variances for the components of output

There are several points that need to be addressed in understanding how the error variances are set for our experiments.

First, while the cyclical properties of output will be affected by the feedback rule from the monetary authorities, we ignore this in the calibration. We set the variances such that a given proportion of the variance of the log-change in output, ignoring the feedback, will come from the permanent and transitory components. That is, the basic system is written as follows:

$$Y_t = YP_t + YC_t, \tag{10}$$

$$YP_t = YP_{t-1} + \varepsilon_{P,t}, \tag{11}$$

$$YC_t = \lambda_1 YC_{t-1} - \lambda_2 YC_{t-2} + \varepsilon_{C,t}. \tag{12}$$

Since YP has a unit root by assumption, we define the relative contribution of the shocks with respect to first differences of the variables,

$$\sigma^2_{\Delta Y} = \sigma^2_{\Delta YP} + \sigma^2_{\Delta YC}. \tag{13}$$

We begin with a value for the overall variance; for these experiments we standardize on a 1 percentage point error variance for overall output growth. The variance for the supply shock follows immediately. Suppose that we want to have a proportion, δ, of this variance coming from the supply shock. Since YP has no other dynamics, we simply set $\sigma^2_{\varepsilon p}$ to δ. The determination of the appropriate variance for ε_c is more complicated. We want the variance of the *change* in YC to be $(1\text{-}\delta)$, but we must transform this into a value for YC itself, and then, because of the autoregressive structure of the YC process, transform this again into a value for the variance of the shock term ε_C.

It is convenient to use the first two autocorrelations of the YC process, written in terms of the parameters: $\rho_1 = \lambda_1/(1\text{-}\lambda_2)$, and $\rho_2 = \lambda_2 + \lambda_1{}^2/(1\text{-}\lambda_2)$. We then have immediately from equation (12):

$$\sigma^2_{\varepsilon C} = \sigma^2_{YC}(1 - \lambda_1\rho_1 - \lambda_2\rho_2), \tag{14}$$

or
$$\sigma^2_{\varepsilon c} = \sigma^2_{YC}(1+\lambda_2)\,(1-\lambda_2-\lambda_1^2/(1-\lambda_2))\,. \tag{15}$$

To complete the system we need the link between the variance of ΔYC and the variance of YC. Writing

$$\Delta YC_t = (\lambda_1 - 1)\,YC_{t-1} - \lambda_2 YC_{t-2} + \varepsilon_{C,t}, \tag{16}$$

multiplying by (YC_t - YC_{t-1}) and taking expectations, we obtain

$$\sigma^2_{\Delta YC} = \sigma^2_{YC}((\lambda_1 - 1 - \lambda_2)\,\rho_1 - \lambda_1 + 1 + \lambda_2\rho_2) + \sigma^2_{\varepsilon c}. \tag{17}$$

Substituting equation (14) and simplifying we find

$$\sigma^2_{\Delta YC} = 2\,(1-\rho_1)\,\sigma^2_{YC}, \tag{18}$$

or
$$\sigma^2_{\Delta YC} = 2\,(1-\lambda_1/(1-\lambda_2))\,\sigma^2_{YC}. \tag{19}$$

Therefore, knowing the desired value for $\sigma^2_{\Delta YC}$, we can use equation (18) or (19) to solve for σ^2_{YC}, and then equation (14) or (15) to deduce the required variance for ε_C.

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