Uncertainty, Learning and Policy Credibility

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Introduction

Monetary policy, its implementation and its effect on economic activity are influenced by many kinds of uncertainty. One aspect of uncertainty that is important for private agents is uncertainty about the objectives of the monetary authority and the conduct of monetary policy, now and in the future. This, in turn, affects the monetary authority because the nature of expectations and the response of expectations to economic developments are a key element in the overall dynamics of the economy.

Historical data from all the industrialized countries show periods of low inflation as well as periods of moderate or high inflation. It seems clear that it is appropriate to characterize history as being made up of different regimes with quite distinct properties, rather than as a sequence of drawings from a single distribution with a rather large variance. Changes in inflation regimes can sometimes be identified with particular events such as the oil price shocks of the 1970s. However, inflation did not respond in the same way in all countries to these shocks. For example, Japan and Germany were able to absorb the shocks without long-lasting effects on inflation, while Canada and the United States were not.

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Most industrialized countries have been able to show a marked improvement in inflation performance in the 1980s, but at different costs in terms of unemployment and foregone output. The conclusions we draw from this aspect of historical experience are that it can be very difficult to wring inflation out of the system once it has become entrenched in expectations, and that the nature of inflation dynamics in an economy does seem to depend on perceptions of the reliability of the nominal anchor, which seem to depend, in turn, on how policy was conducted in the past.

Evans and Wachtel (1992) present evidence that, over long time periods, the process describing U.S. inflation has not been stationary. At the same time, inflation expectations derived from survey data demonstrate systematic errors when evaluated as forecasts. Both inflation and inflation expectations are very persistent. Evans and Wachtel argue that an appropriate way to model this behaviour is by postulating exogenous regime shifts and agents who learn about the current regime by combining historical information with beliefs about the future. Their analysis shows that we cannot look only at the current policy regime to explain the impact of monetary policy. We must also consider past regimes and the policy alternatives. This brings us to the issues of credibility – the public's beliefs concerning the current and future course of policy – and learning – the use of past and current data to predict the future course of inflation and to revise beliefs about the policy regime.

There is an extensive body of literature on learning models. For example, Marcet and Sargent (1989) and others have investigated the convergence of least-squares learning mechanisms to rational-expectations equilibria. In this literature, however, the fact that agents assume that there is one regime and that equilibrium is characterized by rational expectations, leads to dynamics generated solely by the exogenous forcing processes. With learning about the regime, dynamics also arise because agents are slow to change their positions or wait for additional information to confirm or deny their beliefs about the policy regime. Important dynamic effects can also arise from errors in interpreting shocks as changes in policy regime.

Our definition of credibility is similar to the one used by Cukierman and Meltzer (1986). They offer a simple model of a stochastically changing monetary regime where the current regime is unknown to private agents. Agents learn about current monetary policy based on observations of money growth. Credibility is defined in terms of how close private agents are to a correct view on what the regime is. The degree of sluggishness in agents' expectations is associated with the credibility of current monetary policy. This analysis is interesting, but it focusses on the properties of equilibria and most of the key results are determined by exogenous factors. For example, the behaviour of private agents, especially how they respond to the data, is determined by parameters of the system and there is little the monetary authority can do to change the process. If inflation can be tightly controlled, then a change in policy can be quickly and clearly signalled and private agents move quickly to build the change into their behaviour. If inflation cannot be so tightly controlled, agents have a more difficult time separating shocks from changes in regime and convergence is much slower. The main reason why things are predetermined in this sense in the Cukierman-Meltzer analysis is that all agents are presumed to form fully model-consistent expectations, subject to the limited information they have about the goals of the monetary authority.

We have extended the literature in a number of ways. There are two parts to our contribution: a formal "learning" model of expectations formation and simulations of a macroeconomic model in conjunction with this learning model. In the simulations, we focus on the transition to a stochastic equilibrium when a monetary authority chooses a new regime with lower inflation than has been observed in recent history. The nature of the learning process and the development of the credibility of the new regime are front and centre in our analysis. We also discuss how the nature of macro dynamics changes endogenously as learning takes place and credibility in the new regime is developed.

Our learning model uses a Markov switching representation of a multi-state environment. In this representation, there are alternative possible states of the world (policy regimes), each with its own average inflation and dynamic properties. By assumption, agents using such a representation of the data do not know the true regime. They react to the outcome for inflation and assign probabilities to the states. Their expectations are formed using the probability-weighted forecasts from the models for each state. The expectations evolve through time based on a combination of backward-looking, least-squares learning and Bayesian updating of beliefs about the credibility (probability) of the possible alternative regimes.

We estimate such a model using Canadian data and examine its testimony as to how expectations of agents using such a framework might have evolved over the past 40 years. This analysis also provides the starting point for our simulations of a transition to a new low-inflation regime.

In our simulations, we use the Markov switching framework to model the expectations of agents who are "uninformed" about the monetary regime and are content to use simple, albeit state-dependent, autoregressive predictors. Our simulation model also includes "informed" agents who form forward-looking, model-consistent expectations. The informed agents have the same model used by the monetary authority when deciding on instrument settings. Both the informed agents and the monetary authority know that there are uninformed agents and their behaviour takes into account the current forecasts from the Markov model.

With this approach, a certain degree of consistency arises as the uninformed agents learn about the policy regime. However, while these agents may end up assigning "Probability 1" to the true regime and expecting the correct level of inflation on average, they continue to use a "naive," rule-of-thumb forecasting rule and do not form fully model-consistent expectations. Whether this is economically "rational" depends on factors such as the costs of getting better information and making decisions relative to the gains from doing so. Our reading of the various types of empirical evidence is that our approach, with its endogenous response to events and gradual approach to partial rationality in expectations, provides a modelling framework with considerable verisimilitude.

An attractive feature of our approach is that it allows time-varying credibility. The framework permits us to study transitions between policy regimes, the main focus of our simulations, but it also provides a way to characterize the response of agents to shocks and possible non-linearity associated with variation of the credibility of a given regime. Perception of a regime shift need not be caused by a change in the monetary authority's objectives. Unexpected shocks may reduce the credibility of a policy regime that is truly still being followed.

Moreover, unless the effects of such shocks are countered by the monetary authority in a way that signals that the regime has not changed, the loss of credibility is likely to build. Our approach allows us to show how this can happen and to document its importance in the overall problem facing a monetary authority.

In our characterization of the monetary control process, two features are essential to the story. First, the world must be viewed as stochastic, subject to unpredictable shocks whose effects cannot be easily disentangled from changes in the underlying objectives of monetary policy. This is why the word "uncertainty" appears in the title.[1] Second, there must be imperfect monetary control. If the monetary authority could offset the effects of current and recent shocks and sustain a stable outcome, the information problem for private agents would become relatively trivial.

In short, then, we provide a learning model of expectations formation, designed for use in a stochastic environment where the true policy regime is not known by all private agents. We use this model in conjunction with a macro simulation model to study the process of transition to a new regime with lower inflation than has been observed in recent times. We focus on the process of evolution for credibility in the new regime, but we also study the changing nature of nominal dynamics as expectations evolve. Our results indicate that the state of credibility can have a major effect on the interpretation of data and on the properties of nominal dynamics in an economy.

1. The Markov Switching Model of Expectations and Learning

This section describes the Markov switching model (MSM) that we use to characterize learning. The MSM is a way of modelling particular forms of non-stationarity in economic time series. The MSM takes all or some of the parameters

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1. There is a substantial body of literature where uncertainty about future monetary policy is combined with random changes in the actual regime. Cukierman and Meltzer (1986) and Ball (1992b) are two examples. Our learning model is motivated by the same line of thinking, but for the simulations in this paper there is one, fixed policy regime in the future. In effect, we study both the effects of a once-and-for-all change in the policy regime and of the consequent elimination of one element of the uncertainty of the past.

of a reduced-form model as being dependent on an unobserved state variable. In our application, the state variable is the policy regime. The state variable can take on a finite number of discrete values. The sequence of state values is generated by a stochastic process based on transition probabilities between states. The dynamics of the economic variable then depend on the outcome for the state variable. Within each state, the dynamics may be stationary, but when examined over the entire time period they may be (*and* they are in our application) quite non-linear. Moreover, in our particular application, we have found it useful to let one state be formally non-stationary (a unit-root state).

The MSM has some appealing features for modelling expectations formation. In our simulations, the MSM is *not* the true data-generating process. However, we use the MSM to represent how agents who do not know the true data-generating process might form expectations. The advantage over other representations of expectations formation is that we can depict explicitly the process of learning about which regime is operative, without being bound by the restriction that agents eventually converge on literally model-consistent forecasts.

Many economic variables are known to exhibit different time-series properties during different periods of history. Some early applications of the MSM, including work by Hamilton (1989), who popularized its use, have been in the study of business cycles, where expansions and contractions are associated with states with high and low growth rates.[2] Much of the literature has focussed on the ability of the model to assign conditional probabilities to the alternative states. Thus, the model estimates can be used to date the periods of expansion and contraction in an economy. The advantage of this approach is that the data determine the turning points. Allowing time-varying transition probabilities also provides variable conditional expectations of the duration of a contraction or an expansion. Transitions may vary with the duration of a state or they may be associated with other information in the data. Durland and McCurdy (1992) and Filardo and Gordon (1993) have taken different approaches to extending the model in this way.

2. For a review, see Boldin (1992).

Another application of the MSM is in modelling processes that exhibit periodic discrete shifts in regime, where agents' expectations can play a major role in determining outcomes. These include exchange rate, interest rate and inflation rate processes. Hamilton (1988) applied the MSM to a study of the term structure, Lewis (1989a,b) has examined various applications to exchange rates, and Evans and Wachtel (1992) have applied the model to U.S. inflation rates and short-term interest rates. Our work falls into this part of the literature.

The importance of regime shifts for expectations formation can be illustrated by examining the pattern of forecast errors when regime shifts must be taken into account. If a regime shift has occurred in the recent past, but because of noisy signals agents are unaware or not sure of it, then their expectations for current and future variables (which are dependent on the state) will be biased towards the regime previously thought to be in force. If agents believe that a regime shift may occur in the future, then to the extent that they take that possibility into account, their expectations will be biased away from the current regime. Biased forecasts of inflation and exchange rates, which can be found in survey data, may reflect such behaviour. It is important to note that such *ex post* biases do not necessarily imply irrationality. This is clearest for longer-term contracts. For example, in deciding what interest rate is acceptable on a 20-year bond, a purchaser will have to make some assessment, whether explicitly or not, of the probability of future changes in the monetary regime. Even with complete confidence that the current regime is a low-inflation regime, a rational agent may give some weight to alternatives over longer horizons.

The basic MSM specifies a set of possible states and the dynamics for the economic variables within each state. The model can be written as follows:

$$Y_t = X_t \cdot \beta(S_t) + \varepsilon_t(S_t) \qquad \varepsilon(S_t) \sim N(\mu(S_t), \sigma(S_t))$$
$$Pr(S_t = j \mid (S_{t-1} = i)) = \Phi(X_t \cdot \alpha) \quad i, j \in (1, 2, \ldots n) \quad (1)$$

where Y_t is a vector of endogenous variables, X_t is a vector of exogenous variables, S_t is the state variable, and Φ is the standard normal cumulative density function. In its most general form, all of the parameters of the model can depend on the outcome of the state, and the transition probabilities between states can be time-varying, as in Filardo and Gordon (1993), Ghysels (1993) and van Norden and Schaller (1993), depending on variables that are uncorrelated with the state. In practice, however, it is common to limit the number of parameters to make estimation and inference less difficult. Hamilton (1989), for instance, uses an autoregressive structure that is constrained to be the same across states. Only the mean of the growth rate of output (in Hamilton's application) and the standard deviation of the random shocks differ across states. Moreover, most researchers have limited the model to two states.

In this research, we allow for three states, each of which can have a distinct autoregressive structure for inflation. We have not, however, extended the model to include time-varying transition probabilities. This means that only information on inflation is used in forming expectations about inflation. Our final estimated model included a unit-root process and two AR(1) processes, one of which has an unknown mean and autoregressive structure that we estimate, and one of which has coefficients imposed such that it has a long-run average rate of inflation of 1 per cent.

This last restriction was determined by the types of policy experiments we wished to consider in our simulations. Including it in the estimation provides a measure of its credibility from an historical perspective. In particular, we get an estimate of the starting conditions for simulations in which the monetary authority chooses a low-inflation regime. We then use the estimated Markov switching model to study the resulting scenarios as agents learn about the new regime in a stochastic environment.

Letting π represent the rate of inflation, the three-state model can be written as:

$$\pi_t = (1-\rho_1) + \rho_1 \cdot \pi_{t-1} + \varepsilon_{1t} \quad \varepsilon_{1t} \sim N(0, \sigma_1) \quad S_t = 1 \quad (2.1)$$

$$\pi_t = \gamma_2 + \rho_2 \cdot \pi_{t-1} + \varepsilon_{2t} \quad \varepsilon_{2t} \sim N(0, \sigma_2) \quad S_t = 2 \quad (2.2)$$

$$\pi_t = \pi_{t-1} + \varepsilon_{3t} \quad \varepsilon_{3t} \sim N(0, \sigma_3) \quad S_t = 3. \quad (2.3)$$

In general, the transition probabilities between the states are given by the matrix P:

$$P = \begin{bmatrix} p & (1-p)\cdot\lambda_p & (1-p)\cdot(1-\lambda_p) \\ (1-q)\cdot\lambda_q & q & (1-q)\cdot(1-\lambda_q) \\ (1-r)\cdot\lambda_r & (1-r)\cdot(1-\lambda_r) & r \end{bmatrix}. \quad (3)$$

However, in the final estimated model, we restrict the parameters of the transition matrix by setting $\lambda_p = 0$, $\lambda_q = 0$ and $\lambda_r = 0.5$. Thus, the probability of remaining in State 1 is equal to p, while the probability that the regime will shift from State 1 to the unit-root regime, State 3, is $1-p$. Similarly, the probability of remaining in State 2 is equal to q, while the probability that the regime will shift from State 2 to State 3 is $1-q$. Note that under these restrictions, there is no weight assigned to direct jumps between States 1 and 2. This specification treats the unit-root regime as an intermediate state between the other two autoregressive processes as well as a possible description of more extreme experience. It greatly limits the number of parameters to be estimated while preserving flexibility in the transitions between states. The restrictions do not prevent agents from simultaneously changing the weights given to States 1 and 2, but notionally this happens in conjunction with a move through the unit-root state. Transitions out of the unit-root regime into either State 1 or State 2 are assumed to be equally probable, $(1-r)/2$, where r is the probability of remaining in a unit-root state.

A three-state model with different autoregressive parameters in each state makes estimation more difficult because the boundaries between the state processes change depending on the region of (π_t, π_{t-1})-space in which succeeding values of inflation occur and depending on the estimated standard deviation of the errors for each process. Based on the historical data, the estimated standard error for the unit-root state is almost four times that of the state with average inflation of 1 per cent. This makes the unit-root model the natural choice when new data differ sharply from recent experience and agents are unsure of whether there is a stable anchor for inflation or are unsure of which stationary regime is operative.

The MSM is estimated by maximizing the likelihood of the observed values of inflation over the unobserved states. Each outcome of the unobserved state implies a particular distribution for the observed values of inflation. By specifying the distribution of inflation in each of the states and the distribution of the unobserved state variable which, in our case, is determined by the fixed transition matrix, a joint conditional density function can be obtained for each state. Summing over all states yields a likelihood function that can be maximized with respect to the parameters of the system.

The estimated MSM is used in our simulation model to form expectations of current and future inflation given the past history of inflation. Expectations in an MSM must include conditional probabilities of the current and future states. These probabilities form part of the output of the estimation of the model. For example, for a data set of size T, the maximum-likelihood estimation routine[3] supplies $Pr(S_T = i | \pi_T, \ldots \pi_1)$ for $i = 1, 2, 3$. The conditional probabilities of States 1 and 2 for period $T+1$ are then given by:

$$Pr(S_{T+1} = 1 | \pi_T, \ldots \pi_1) = Pr(S_T = 1 | \pi_T, \ldots \pi_1) \cdot p + [1 - Pr(S_T = 1 | \pi_T, \ldots \pi_1) - Pr(S_T = 2 | \pi_T, \ldots \pi_1)] \cdot (1 - r)/2 \ . \quad (4.1)$$

$$Pr(S_{T+1} = 2 | \pi_T, \ldots \pi_1) = Pr(S_T = 2 | \pi_T, \ldots \pi_1) \cdot q + [1 - Pr(S_T = 1 | \pi_T, \ldots \pi_1) - Pr(S_T = 2 | \pi_T, \ldots \pi_1)] \cdot (1 - r)/2 \ . \quad (4.2)$$

The probability that the current state is State 1 is given by the conditional probability that last period's state was 1, times p (the probability of remaining in

3. The programs used in estimating the models are based on Hamilton's code, with modifications by Simon van Norden and the authors.

State 1), plus the conditional probability that last period's state was 3, the unit-root state, times $(1-r)/2$ (the probability that there was a transition from State 3 to State 1). The formula for State 2 has the same structure, with q substituted for p. The probability assigned to State 3 can then be determined residually. The one-period-ahead forecast is then given by:

$$E[\pi_{T+1}|\pi_T,\ldots\pi_1] = Pr(S_{T+1} = 1|\pi_T,\ldots\pi_1)\cdot(\gamma_1+\rho_1\cdot\pi_T)$$
$$+Pr(S_{T+1} = 2\,|\pi_T,\ldots\pi_1)\cdot(\gamma_2+\rho_2\cdot\pi_T)$$
$$+[1-Pr(S_{T+1}=1|\pi_T,\ldots\pi_1)-Pr(S_{T+1}=2\,|\pi_T,\ldots\pi_1)]\cdot\pi_T. \quad (5)$$

Forecasts for periods farther into the future can also be computed by iterating forward with this formula. These expectations converge as the conditional probabilities converge.[4]

Within the simulation model, as new information in the form of observations of inflation is processed by agents, the probabilities and the standard errors in the Markov model are re-estimated. Initially, high variability in the data can work to the disadvantage of the 1 per cent rule because, as indicated, these data are more likely to be assigned to the unit-root state. But, as the monetary authority continues to pursue the low-inflation target, two things happen. Assuming that the monetary authority is successful enough in keeping inflation around the target, agents will begin to assign higher probability to the true state. At the same time, the estimated standard error for the 1 per cent state will rise towards the observed *ex post* variability of inflation. Thus, as credibility develops, the probability that random shocks will lead agents to jump to another view declines. We show that this point is of great importance to the overall nominal dynamics of the economy.

4. The conditional probabilities can be shown to converge to $(1-q)(1-r)/2s$ for State 1 and $(1-p)(1-r)/2s$ for State 2, where $s = (1-p)(1-q)+(1-q)(1-r)/2+(1-p)(1-r)/2$.

2. A Markov Switching Model of Inflation Expectations in Canada

2.1 An interesting failure

We experimented with a number of formulations of the Markov framework. We tried differing numbers of states and various forms of restrictions on the parameters to aid the identification. We wanted the same general framework to work reasonably well for all G-7 countries.[5] This ruled out some interesting results for particular countries, but it is difficult to profess confidence in a model and an interpretation of history for Canada, say, that does not seem to work in roughly the same way for the United States.

Before describing the version adopted for this research, we want to review briefly one rejected model. We tried to find a formulation that would estimate a "high and stable" inflation regime as one of the alternative states. We were not successful. The results were very unstable, in several senses. In some cases, the estimated mean would drift off to ever-higher values; the estimator could not distinguish the mean from the adjustment process. In other cases, the estimator found a solution, but we were unable to give a reasonable interpretation of the results, either for the specific country or consistently across the countries. We tried to make this formulation work because we wanted to see if we could find an alternative to a unit-root specification for the periods of high inflation. We think that it is interesting that we could not identify a regime of high and stable inflation.

2.2 The model for Canada

For this study we measure prices and inflation using the Canadian gross domestic product (GDP) deflator.[6] Table 1 (see Appendix for table and figures) shows the maximum-likelihood estimates of the main parameters. The estimated mean inflation rate for State 2 is just over 3 per cent. Thus, the three states

5. We have estimated a model of the same type for each of the G-7 countries. This research will be reported in a separate paper.
6. We have also estimated the model using the consumer price index. The results are similar but not identical.

considered by agents in the estimated model are: i) 1 per cent inflation with relatively speedy mean reversion (imposed); ii) 3 per cent inflation with a moderate rate of mean reversion (estimated); and iii) a unit-root world, where the unconditional mean inflation rate is undefined and inflation follows a random walk. Note that the parameter α_1, and hence p (the probability of staying in State 1) and σ_1 (the standard error for State 1) are not as well determined as the parameters for State 2. This is not surprising; as we shall see next, State 1 is not assigned much weight in the historical sample. Note also that the point estimates of the transition probabilities all show some degree of regime stability. That is, given that we are in State i, the probability assigned to staying in State i is not absolutely small for any state. It is lowest for State 1 and highest for State 2. Note, finally, that the estimated variability of inflation is higher in State 2 than in State 1, and much higher in State 3 (the unit-root state) than in State 2. Given that State 3 tends to be associated with periods of high inflation, the estimates show a strong association between the level and variability of inflation.

The estimated state probabilities and the resulting expectations are shown in Figure 1 (see Appendix for figures). At first glance, the expectations look much like those one would obtain from autoregressive formulations. However, it is hardly surprising that we see an important element of response to the data in any representation of expectations. It is essential to look beyond the gross historical correlations and focus on particular episodes. There are a number that deserve attention. In the run-up of inflation in the early 1970s, we see a significant lag in the response of expectations. There is also a lagged response to the escalation in the late 1970s and early 1980s that extends the adjustment after the recession into 1984-85. The end of the sample is also very interesting. Expectations do not follow the actual rate down into the region of 1 per cent inflation. This is quite different, at least so far, from the results in the mid-1980s when inflation returned to a historically average level. Let us look more closely at the story behind these numbers in the Markov probabilities.

In the period up to 1962, the estimates tell us that there was some uncertainty about the underlying state.[7] The highest weight is assigned to the 3 per cent state. It is interesting, however, that the low, 1 per cent state is given some weight, except in 1956, and a considerable weight in 1961. This measure of inflation dipped below 1 per cent in 1961 as it had once before in the sample (1955). On the other hand, agents also put some weight on the unit-root state, with a brief spike to a relatively high weight in 1956. The unit-root model acts as an intermediate state, a place to stand when there is confusion about the underlying mean. When the rate of inflation changes relatively sharply, the model tends to recognize the uncertainty created by raising the weight given to the unit-root state until the situation becomes clearer.

There is a notable change beginning with the recovery from the 1961 recession and into the period when the Canadian dollar was pegged to the U.S. dollar. As inflation escalates, agents lose faith rapidly in the low-inflation state. Gradually, confidence develops in the regime where inflation averages about 3 per cent. Through the last half of the 1960s and into the early 1970s, this state is given high probability. Note, however, that the unit-root state is never totally discarded. The probability assigned to it drifts down very slowly over the decade and settles at about 0.1 for about 4 years.

With the refloating of the Canadian dollar, the loosening of monetary conditions in 1972 and the oil price shock of 1973, inflation escalates rapidly. The credibility of the stable inflation policy is lost equally rapidly. Within 2 years, agents have abandoned the model of the 1960s and switched to the unit-root model where there is no stable inflation rate and no credibility in the monetary authority's commitment to a nominal anchor. As inflation declines again, agents shift back towards the old model. It takes longer to restore credibility than it took to lose it, but there is a jump back. One could make a case in support of the view that the Anti-Inflation Board had an effect on the underlying model, based on the delayed

7. There is some arbitrariness in the results for the first few periods in a Markov switching model because starting values must be provided. We have used the full-sample estimates of the unconditional probabilities of the states to start the sequence. Nevertheless, the results for the 1950s must be treated as less robust than those for the rest of the sample.

final increase in the weight assigned to State 2. Whatever the reason, according to our estimates, uninformed agents became convinced that inflation was heading back down to the levels of the 1960s, despite the fact that it never came close to doing so. This may help explain why *ex post* real interest rates appear unusually low through this period.[8]

As inflation surges again at the end of the 1970s, credibility evaporates very quickly indeed and agents jump back to the unit-root view. This time it takes much longer for agents to become convinced that there is, in fact, a nominal anchor. According to these results, there may have been some slight recollection of the earlier low-inflation world during 1983-1984 when inflation came down dramatically. This suggests that there was a chance at that time to begin a transition to a credible policy of low inflation. However, when no further progress towards price stability was forthcoming, this chance was lost, and agents settled on a 1960s' view for the rest of the decade and into the 1990s.[9] Moreover, as inflation by this measure drops markedly through 1990-1992, there is little response in the agents' characterization of the process, or of the expected values themselves. Only in 1992 do we see the first signs of emergence of any credibility for the low-inflation world as the probability assigned to State 1 edges up to just under 0.1.

In Figure 2, we show the data from 1975 to 1992, adding for comparison the average expected inflation as revealed by the survey of economic forecasters carried out by the Conference Board of Canada. The data here are the average one-year-ahead forecasts from the autumn editions of the Conference Board's *Canadian Business Review.*[10] One would expect that economic forecasters would be somewhat better informed than the general population, in particular those

8. As we noted above, the standard error associated with the unit-root state is also relatively large. As this model tends to be associated with periods of high inflation, our results indicate a correlation between the level of inflation and the degree of uncertainty about inflation. This result may also prove useful in discussing the evolution of the term structure of interest rates. Our result can be interpreted as a formalization of the argument in Ball (1992b).
9. There is no evidence of an announcement effect from the Hanson lecture (Crow 1988) or the establishment of the inflation-reduction targets. However, this model cannot show announcement effects unless there is some change in the actual outcome at the same time.
10. The data are described and listed in Laxton, Rose and Tetlow (1993a). We have no data for the period prior to 1975.

agents whose expectations we wish to describe with the MSM. We see some evidence of that in the late 1970s. However, the evidence from the 1980s is much less clear. Unfortunately, we do not have enough survey data to see what happened in the period of escalating inflation in the early 1970s, so it is difficult to draw any strong conclusions. Nevertheless, it is interesting that our "uninformed" agents would have outperformed the economic forecasters in forecasting inflation through most of the 1980s.

3. The Macro Model

We use a simple model of how the economy functions and how monetary policy works at a very high level of abstraction. The model describes four aspects of the macro economy: inflation, inflation expectations, output and the policy control process.

Inflation is driven by a mixture of expectations and forces based on market conditions. Demand conditions play an important role in the determination of inflation, but expectations of inflation are themselves, to a point, an independent determinant of inflation. Hence, there are potentially two channels of effect for monetary policy: through output gaps and through expectations.[11]

This depiction of the determinants of inflation is represented in our stylized model by equation (6), where $YGAP$ is the output gap and where Π and Π^e are inflation and expected inflation[12]:

$$\Pi_t = 0.2 \cdot \Pi_{t-1} + 0.8 \cdot \Pi_t^e + 0.315 \cdot YGAP_{t-1} + \varepsilon_t. \qquad (6)$$

11. The influence of monetary policy through the exchange rate adds another relative price dimension to the general discussion; we do not deal explicitly with open-economy issues in this paper.
12. Laxton, Rose and Tetlow (1993b) study the properties of a similar economy with a non-linear Phillips curve. Given the empirical support for a non-linear specification reported in Laxton, Rose and Tetlow (1993a), it would be interesting to extend this research to include non-linearity in the economy. This would make the establishment and maintenance of credibility in a stochastic economy all the more challenging.

Equation (6) has an inertia term included to pick up the effects of intrinsic propagation of inflation from contracts, habits, trading relationships and so on. Dynamics coming from expectations are added to this. We consider only the case where there is no permanent trade-off between inflation and output.[13]

A key part of the uncertainty in the analysis comes from shocks to inflation. The error term, ε_t, is assumed to be normally distributed with variance 0.81.[14] The feature of our model that makes these shocks especially interesting is that expectations respond endogenously to the outcome through the Markov state probabilities. This means that there is a potential for discrete jumps in expectations if the inflation outcomes lead agents to change their underlying characterization of the monetary regime.

Equation (7) is our representation of expectations formation in the simulation model:

$$\Pi^e_t = 0.7 \cdot \Pi M_t + 0.3 \cdot \Pi^e_{t,\,t+1}. \tag{7}$$

We model expectations of inflation as a linear combination of the predictions of the Markov switching model and the economic simulation model. We think of this linear combination as reflecting the behaviour of the two types of economic agent. The model-consistent expectations reflect expectations of informed agents. In this case, being "informed" includes knowing the true goals and behaviour of the monetary authority. For these experiments, we fix the

13. The calibration is based on estimates reported in Laxton, Rose and Tetlow (1993a). However, we have lowered the coefficient on the intrinsic part of the dynamics. The coefficient used here is closer to what one gets from a "contracts" view of this term. It is not surprising that it would be higher in our estimated model, which used the forecasts of modellers taken by the Conference Board as the measure of expected inflation. From the perspective of this paper, one might suppose that the economic forecasters are relatively informed. This would mean that part of the coefficient on the lagged inflation rate in the estimated model would be attributable to the expectations of uninformed agents and not truly intrinsic dynamics.

14. This number should be interpreted as squared percentage points. It is based on the error variance from estimates of equation (6). We have lowered the value somewhat, relative to the estimated variance, on the grounds that specification error and policy changes are likely to have contributed to unexplained variation in inflation, historically.

proportion of informed agents at 30 per cent. The Markov expectations are characterizations of forecasts made by uninformed agents who observe inflation and make simple autoregressive projections based on their characterization of the probabilities assigned to the alternative regimes. The model has endogenous learning by the uninformed agents in the form of Bayesian updates of these probabilities as new data emerge. Understanding this process is a key objective of our paper.

The third aspect of the model concerns the properties of the output cycle and the way that policy affects output. An accepted stylized fact is that shocks to aggregate demand have persistent effects, reasonably described by a low-order autoregressive process. Moreover, the impact of a change in the policy instrument is not felt immediately; we assume that the order of this lag is about the same as the order of the autoregressive process for output. For our purposes here, the policy instrument, R, can be thought of as any control variable. These relationships are shown in a stylized "IS curve," equation (8):

$$YGAP_t = 0.61 \cdot YGAP_{t-1} - 0.8 \cdot R_{t-1} - 0.2 \cdot R_t + \eta_t. \tag{8}$$

The control variable, R, is measured relative to an equilibrium level that is not determined in this simple model. Similarly, the equilibrium level of output is not determined within the model and is assumed independent of the monetary regime and of the particular shocks that arrive. Finally, note that the IS curve includes a stochastic disturbance term, η_t. We assume that this shock is also normally distributed and that it is independent of the inflation disturbance. We set its variance at 0.79. This number comes from an estimated equation like equation (8). It is the value used in Laxton, Rose and Tetlow (1993b).

The simple model sketched above is best thought of as an annual model; a quarterly representation would require more lags and more complicated representations of how dynamic processes interact. Of course, in the real world, the monetary authority often gets considerable information about shocks in the year they occur. This is offset, however, by the fact that the more complete model would have a somewhat longer lag for full influence of the control variable on output. We do not think that the details are important for this analysis. What is important is that demand shocks have autocorrelated effects on output and affect inflation with a lag and that the monetary authority can have only a small contemporaneous effect on output and inflation. The control process works with an important lag.

The delayed effect of the policy instrument on output is one reason why it is not appropriate for a monetary authority to aim to keep inflation precisely on target in a stochastic environment. If the monetary authority cannot know the future shocks it cannot, except by chance, hit the inflation target precisely. Moreover, the lags in the effect of policy and the other sources of inertia in the system limit the speed at which it is reasonable to try to bring inflation back towards the target following a shock. However, the lags also mean that it is very important for the monetary authority to anticipate the course of the economy that is suggested by the starting point and the equations of motion of the economy. In other words, the control rule must be forward-looking.

The delayed effect of the policy instrument is also important to the motivation of the learning problem itself. Because the monetary authority cannot neutralize the effects of shocks, the uninformed agents cannot know from observing a change in inflation whether they are observing the effects of shocks or a change in the policy regime or both. The problem would not be interesting if every variation in inflation signalled a deliberate change in policy.

One of the questions we address in our simulations is the nature of the policy rule that is required to control a stochastic economy when the monetary authority wishes to move to a new inflation regime and to establish and maintain credibility for that new policy. This issue is one where the techniques of optimal control could be very useful. However, we are not yet able to implement an optimal control technology for this problem. Instead, we focus on simple rules and on finding the features of a rule that are linked to the model of expectations. Because we are investigating transition to a new regime that is outside most historical experience, it is interesting to ask whether this has any special implications for the transitional stance of monetary policy. Moreover, because we are introducing the notion of regimes explicitly into expectations, with the consequent risk of fragility of credibility, it is interesting to explore this issue and ask whether the risk of jumps in expectations and loss of credibility for the desired regime suggests any modifications in the conduct of monetary policy, either in the transition period or in the new stochastic steady state. Indeed, the issue of whether or not the monetary authority can achieve full credibility in a new regime in a stochastic environment, and if so, how long it might take, are all questions we address.

In considering these issues, we have experimented with a variety of modifications on the control rule. We will provide further details with the simulation experiments themselves. For now, the important point is that the model is always closed with a forward-looking control rule based on the following starting point:

$$R_t = \delta \cdot \left[\sum_{i=1}^{3} (\lambda_i \cdot \Pi GAP^e_{t,t+i}) + \lambda_4 \cdot YGAP \right] + (1-\delta) \cdot R_{t-1}. \tag{9}$$

The policy rule has two parts. The first term in the square brackets is the policy-targeting part. It triggers response to *expected* future gaps, $\Pi GAP^e_{t,t+i}$, while recognizing that closing these gaps quickly would be costly. The use of *future* inflation gaps reflects the fact that policy can affect output only with a lag. That these are *expected* gaps reflects our simulation methodology, wherein future shocks are not known when the policy instruments are set. The inclusion of *YGAP* could reflect its usefulness as an additional indicator of latent inflationary pressure. Alternatively, we could use this term to implement a rule where the monetary authority gave some weight to stabilizing output in setting the policy instrument.

In previous work,[15] we have used a rule that focussed on the expected deviation of inflation from the target level, usually allowing some difference

15. See, for example, Laxton, Rose and Tetlow (1993b).

between shorter-term targets and the steady-state target during periods of transition. For this work, we have altered the definition of the operating rule in an important way. While the ultimate goal of monetary policy can be characterized as tying the inflation rate to some long-term target level, in a stochastic world, and especially in a world where expectations are formed largely based on incomplete knowledge of the true goals of policy, there is good reason for the monetary authority to take as its proximate objective the anchoring of expectations themselves. As a practical matter, there are problems with such a rule in that we have, at best, imperfect measures of expectations. However, we have the advantage here of an explicit representation of the expectations-formation process and a model that provides both a measure and an understanding of the sensitivity of expectations to the data.

We define the inflation "gap" that the monetary authority uses in determining the settings for the instrument as the difference between the expectations generated by the Markov model and the target rate of inflation. The monetary authority thus works to keep the expectations of the uninformed agents controlled around the policy objective. This rule is perfectly consistent with attaining the desired actual inflation rate. It has the advantage of focussing on an important source of instability in the nominal dynamics.

It is important to understand that the potential for jumping in the Markov expectations creates the potential for powerful non-linearity in the dynamics of inflation. One can easily generate scenarios where the point expectations stay close to a desired level when agents have shifted their view of the regime. The MSM has adjustment dynamics that permit such transitions. But the key point is that once the characterization of the regime has moved, expectations become anchored to the new view and it becomes much more costly to restore the original stochastic equilibrium once credibility has been lost in this way. This is what we mean by the potential for very powerful non-linearity in the expectations process. By focussing on the Markov expectations in the policy rule, we take an important step in controlling these dynamics. Thus, we have:

$$\Pi GAP \equiv \Pi M - \Pi TAR, \tag{10}$$

where the target is a linear combination of the steady-state target and a measure of the current state (to allow for gradual transition to the long-term target). For this work, we have used the following definition of the target:

$$\Pi TAR = 0.1 \cdot \Pi M_{t-1} + 0.9 \cdot \Pi SS, \tag{11}$$

where the SS variable is the long-term target inflation rate which, in this instance, is 1 per cent.

The second part of equation (9) is the R_{t-1} term which represents a desire, on the part of the monetary authority, to avoid excessive volatility in the instrument settings. This desire may reflect uncertainty regarding the controllability of instruments, or concern that measured point elasticities of a model may not be valid over larger variations in instrument settings. Alternatively, it may reflect other implicit objectives or constraints on the monetary authority's behaviour.

4. The Simulation Experiments: An Overview

4.1 The simulation methodology

Combining a forward-looking simulation of a macro model with a Markov-state representation of expectations and learning has not, as far as we know, been done before.

The macro simulation framework is relatively standard, but our algorithm, which has been developed at the Bank of Canada for this type of work, is also relatively new.[16] The techniques used for stochastic simulations do not permit the monetary authority to know the future shocks. The control solutions that determine the instrument settings in each period are full, forward solutions, conditional on knowledge of the predetermined state variables and the current shocks. The policy

16. The algorithm provides an advance on standard algorithms, such as Fair-Taylor, in eliminating type-2 iterations by "stacking" the problem to take time out of the system. It requires a lot of memory, but is much faster and more robust than Fair-Taylor.

settings are thus fully consistent with the model and the initial conditions, but they do not embody perfect foresight.

The complication is that in each period we must compute the updates to the Markov model and the new expectations. This is currently set up as a routine in a Gauss program. For each period, we must first execute the Gauss program to get the expectations of the uninformed agents. The monetary authority and the informed agents then take these as given in the simulation of the macro model for that base period, which gives the outcomes for the period, including the current setting of the policy instrument. However, since the macro model involves a forward-looking control problem, the macro simulation at each point in time involves a complete forward solution. One could call this solution the expectations of the monetary authority and the informed agents. The Gauss program is then run again to update the expectations of the uninformed agents for the next period, and so on.

A limitation of this process is that the macro solution at a point in time does not take into account the revisions of the Markov model that would be expected, based on the forward solution of the monetary control problem at that point in time (that is, before the future shocks are known). The solution at each point in time does take into account the dynamics within the Markov model estimated up to that point, but the Markov model itself, and in particular, the probabilities assigned to each state, are held fixed for each solution of the monetary control problem. To relax this restriction would require iterating on the expected solution of the Markov model for all future periods at each point in time (that is, within each solution of the monetary control problem). This would increase substantially the computational requirements of the exercise. Since the Markov model is, in fact, updated each period, we do not think that our results are compromised by this minor limitation in our methodology.

4.2 Credibility once again: the issues

The historical estimates of the MSM indicate that uninformed agents were assigning low probability to the low-inflation state, at least as of 1992. We turn now to simulation analysis to consider a number of questions about the transition to the new regime and the eventual maintenance of credibility in a stochastic steady state.

Let us first review what we mean by "credibility" in these experiments. We take as a maintained hypothesis that the monetary authority has chosen the low-inflation regime and does actually pursue that goal unwaveringly. The uninformed agents do not know this, however, and the essential task of the monetary authority is to engineer a sequence of outcomes that convince them, through the normal learning process in the Markov system, that there is indeed a new low-inflation regime.[17]

A natural index of credibility in this process is the probability, which we label PS1, assigned by uninformed agents to the possibility that they are in State 1, the low-inflation state. There are other interesting measures of aspects of credibility that can be derived from the Markov framework, such as the expected duration of a state and the effective confidence interval for the probabilities. The latter tells us how much variation in the actual outcome for inflation would be accepted by uninformed agents as random and not sufficient to trigger a reconsideration of the probabilities. In actual fact, the effects of the data are continuous, but there is strong non-linearity in the effects of new observations such that the outcome has jumpy characteristics. Small perturbations do not generate much change in the estimated state probabilities, but the response increases dramatically at some point. We will illustrate this in some of our simulations. While these other dimensions of credibility are of considerable interest, our main focus is on the probability assigned to being in the regime actually being followed by the monetary authority.

Our ultimate goal is to look at the transition to and maintenance of a credible stochastic equilibrium. The fact that there will be shocks, perhaps very unfortunate shocks, is essential to the problem. These shocks create the fundamental uncertainty in the learning environment of the uninformed agents. Suppose, for example, actual inflation rises unexpectedly. What does it mean? Has

17. A very important extension would be to add true uncertainty about the future monetary regime to the analysis. Here we abstract from that aspect of uncertainty, in the sense that there never is a change in regime.

there been a shift in the policy regime or in the operating rules for policy, or is it just that there have been highly inflationary shocks? Eventually, we hope to model the relatively uninformed agents as responding to other relatively costless information and not just the outcome for inflation. For example, it might be interesting to have the uninformed agents look at some other economic information using some rough (perhaps reduced-form) view of the economy, and in particular to look at the policy instruments. Here, however, the uninformed agents consider only inflation in forming their expectations. The monetary authority can lose credibility even when it is fighting against the shocks if the outcome is not sufficiently controlled.

In the stochastic simulations, we construct 60 trials with independent sequences of drawings for the shocks generated using the quasi-random number routines in TROLL. We start the simulations using the historical conditions of 1993. There are a number of issues. First, we investigate the process of expectations on average across the trials to get an idea of whether full credibility is a realistic goal and, if so, how long we should expect that process to take and what the costs might be in terms of the real economy. We also report results for a few interesting individual trials to illustrate the kinds of problems that can arise for a monetary authority in a stochastic world with agents who use Markov learning rules. Second, we examine the influence of credibility on the nominal dynamics and volatility of the economy by studying the effects of inflationary shocks at different points along the transition path. Finally, we look at a number of issues under the general heading of sensitivity analysis. For example, we look at the sensitivity of the results to the assumptions about the stochastic specification.

To start the discussion, however, we report some simpler artificial experiments designed to provide some basic understanding of how the two parts of the model interact. We still begin with the 1993 starting point, but we use deterministic simulation analysis to explore the following three issues. First, to get some appreciation for the degree of rigidity in expectations inherited from history, we ask how long it would take to establish credibility if there were no more shocks. Second, we introduce the issue of the sensitivity of expectations and the volatility of the economy to credibility. Finally, by varying the size of the shock, we illustrate the nature of the non-linearity introduced by switching in the Markov expectations. This is interesting in its own right, but it is especially important as background to understanding the results of the full stochastic simulations that follow.

5. Deterministic Simulations

5.1 The policy rule

For the simulations we use the following policy rule:

$$R_t = 0.90 \cdot [2.0 \cdot \{1 + 1.5 \cdot (\Pi GAP > 1) + (\Pi GAP < 1)\} \cdot \{0.25 \cdot \Pi GAP^e_{t,t+1} + 0.5 \cdot \Pi GAP^e_{t,t+2} + 1.0 \cdot \Pi GAP^e_{t,t+3}\} + \{1 - (abs(\Pi GAP) > 1)\} \cdot YGAP] + 0.1 \cdot R_{t-1}. \tag{12}$$

There are several changes from the introductory discussion in Section 3. First, we have magnified the response when the ΠGAP outcome is more than 1 percentage point from zero. This gives a kind of "band rule," where policy action is made stronger if the outcome has drifted outside some region. Second, we have made this extra response asymmetric – it is larger when there are big positive errors than when there are big negative errors. The asymmetry is in the form of stronger response to shocks that create risks of a jump in expectations (or evidence of such movement) towards the higher-inflation regime. Based on postwar history, the expectations model is structured such that the main risk is a jump in expectations back to higher levels. There is nothing sacrosanct in this structure. If we were to observe regimes with systematic deflation, we would have to change the characterization of interesting states. But, for now at least, we accept the historical starting point as binding on the characterization of the states. Finally, we turn off any concern for the output gap if ΠGAP is outside the "bands."

This policy rule embodies what we have been able to learn so far about what works in establishing credibility in the new inflation regime. To be successful in achieving and maintaining a low-inflation regime, a monetary authority must provide an anchor for inflation expectations. While controlling actual inflation will achieve this end, as an operating rule that misses a key point – that when there are expectations of a regime shift, actual and expected inflation will not jump immediately to the new state because of a recognition by agents that there is inertia in the system. If a monetary authority waits until actual inflation has responded fully and has become entrenched in point expectations, the job of restoring confidence in the regime becomes much more difficult and more disruptive to the economy. Our preliminary work in response to these points has involved various ad hoc modifications to our ad hoc policy rule. Both the non-linearity and the asymmetry of response in the above rule improved the success of the monetary authority in establishing credibility in the new regime.

5.2 The experiments

We report results from the following experiments. First, we have a no-shock base line, which provides the answer to our first question – how difficult should we expect the transition to credible stochastic equilibrium to be, if we start with the conditions of 1993 and abstract from further shocks? We then add shocks of two different magnitudes at two time points. In each case, we shock inflation directly. To illustrate the non-linearity of expectations dynamics we apply shocks of 1 and 2 standard errors (of inflation). To illustrate the importance of credibility to the overall nominal dynamics of the economy, we introduce these shocks first in Year 4 (1997) when the transition to credibility is well under way but still far from complete, and then again in Year 7 (2000) after credibility has been almost fully established.

5.2.1 How long does it take to gain credibility?

Figure 3 (see Appendix for figures) shows some results from the base-case simulation. The solid line shows the probability assigned by the uninformed agents to the possibility of being in the low-inflation regime that is actually being pursued by the monetary authority. From about 0.1 as an initial condition, this probability rises slowly at first and then rapidly to about 0.5 in 1997 and 0.8 in 1999. Thereafter, the pace of convincing the remaining sceptics slows and it takes until

Year 8 (2001) before all agents have fully accepted the existence of the new regime.

The Markov expectations are initially well above the actual rate of inflation and the new target. As the recovery proceeds, actual inflation moves up close to the target by 1997 and expectations, after overshooting the target for a couple of years, jump back up and then converge fairly quickly on the long-term target. [18]

The results one gets in this experiment depend very much on the nature of the initial conditions and the specific profile of the no-shock recovery. Space limitations prevent us from including more experiments, but we can report that a moderate positive inflation shock in the early part of the sample, when inflation is simulated to be below the target, would speed up the transition to full credibility. For example, if inflation were shocked by 1 standard error in 1994, *ceteris paribus*, full credibility is achieved sooner. The short-term inflation outcome generated by the initial conditions is *too low* for agents to accept the supposed low-inflation regime. They are led to assign extra weight to the unit-root world and, paradoxically, the 3 per cent world, than they would if inflation were closer to 1 per cent over the near term.

5.2.2 *The fragility of expectations with imperfect credibility*

Figure 4 shows the results for the 1 and 2 standard error shocks to inflation in 1997 when, in the base case, the credibility index has reached about 0.5. The shocks are for one period only; thereafter, we return to the base-case assumptions.

Note the strong non-linearity in the results. For the smaller shock (which is not absolutely small) there is some loss of credibility, initially, as actual inflation jumps to about 1.8 per cent. However, the effect on credibility is relatively small and short-lived and the overall approach to full credibility is not much different from the base case.

18. We focus on the expectations of the uninformed agents in presenting the results. Recall, however, that the overall inflation expectations are determined also by informed agents with a weight of 30 per cent.

The results for the larger, 2 standard error shock are much more dramatic. Actual inflation jumps to close to 3 per cent in 1997, and this destroys the fragile credibility of the new policy regime despite a vigorous response by the monetary authority. It takes about 4 years to regain the lost credibility and much longer to build complete confidence in the new regime. The non-linearity is also clear in the paths of the policy instrument and the output gap. These are shown in Figures 5 and 6. The policy instrument is measured in percentage points (like a deviation of an interest rate from an equilibrium level). The output gap is measured as a per cent of potential output.

5.2.3 The gains from credibility: the 1 standard error shock

Consider now the smaller shock, a 1 standard error shock to inflation, but now supposing that the shock arrives after a high degree of credibility has been established. Figures 7 and 8 show the results for the credibility index and the output gap for this shock, with the previous results – that is, the same shock in 1997 – also shown for reference.

The results show a dramatic difference. Although the actual inflation rate rises by roughly the same amount in the two instances, the rest of the results are quite different. Once credibility in the low-inflation regime has been established, uninformed agents will tend to interpret the 1 standard error shock correctly and will not change their underlying characterization of policy to any significant extent. This has striking consequences for the monetary authority. There must be some response to the shock, but it is very small compared with what must be done if the same shock arrives in 1997 when confidence in the new regime is relatively fragile. We have not illustrated this directly, but Figure 8 shows the difference in the output gaps necessary to retain control on inflation.

This provides an important tentative conclusion – there may be significant gains in reducing the volatility of the economy and the monetary intervention necessary to sustain a regime once the credibility of that regime has been established. This is a very important point if one accepts the premise of this model that the monetary control process in a stochastic environment is itself an important source of cycles or the propagation of cycles in the economy. There will always be shocks. To the extent that a monetary authority can build credibility in the nominal anchor, the real consequences of these shocks may be significantly attenuated. We pursue this important point further in the next section.

5.2.4 The gains from credibility: larger shocks

Figure 9 illustrates what happens to the credibility index when we double the shock to 2 standard errors. The monetary authority is forced to respond a bit more vigorously. Credibility is reduced initially, but by far less if there is full credibility than if the shock hits earlier in the transition. Even a shock as large as 3 standard errors does not destroy credibility completely. Credibility, once established, apparently provides an extended buffer and reduces the variability of the cycle for a wide range of shocks.

5.2.5 Credibility and non-linearity

Figure 10 compares the response of actual inflation and the Markov expectations for the 1 and 2 standard error shocks under conditions of full credibility. We can see in this figure that actual inflation does react virtually one-for-one with the shock on impact. However, we can also see an aspect of the important non-linearity in this model. The expectations move proportionately more with the shock the greater the decline in credibility of the regime.

6. Stochastic Simulations

6.1 Overview

The real task facing a monetary authority is not well described as one of responding to isolated shocks to a deterministic economy. There are always shocks and they cannot be anticipated. Hence, stochastic simulations provide a more realistic environment for the study of policy issues. In stochastic simulations, shocks are drawn from their hypothetical distributions, and through repeated trials we can construct the distribution of outcomes that might be expected given a policy rule. We will see in the stochastic simulation results that a monetary authority must be prepared for some difficult circumstances.

In our simulations, the problem of gaining credibility is cast in extreme terms. The uninformed agents do not use any other indicator than the actual outcome for inflation in choosing a view about the policy regime. In a stochastic world, this process is inherently much less stable than in the deterministic world. In fact, at this stage of our research we are unable to report results where 100 per cent credibility is achieved, even after 50 years of dogged pursuit of a single policy goal. The policy goal itself is achieved, on average, but the policy regime never becomes 100 per cent credible. We are not yet sure whether this is a fundamental result or a reflection of a policy operating rule that is not quite good enough at responding to the difficult draws.[19] We intend to return to this issue using optimal control techniques. For now, we can report that we were unable to find one rule that would always succeed in gaining full credibility within 50 years. In some cases, we could find a rule that would work for a problem case, but would lead to worse results for some other trials.

Nevertheless, we have some intriguing results to offer. We begin with the average results across the 60 trials. We then report measures of variability across the trials and a few individual trials to illustrate the variety of circumstances that a monetary authority must be ready to deal with. Next, we repeat some of the shock analysis from the previous section in a stochastic mode. Finally, we present a first look at the sensitivity of the results to our assumptions.

6.2 Average results for 60 trials

Figure 11 shows the results for the main variables of interest. We have included a few years of recent history to show the initial conditions for the experiments.

Consider first the middle panel, where we report the probabilities assigned to the three states by the uninformed agents. Recall that in the deterministic world, it took about 8 years to achieve 100 per cent credibility, defined as a probability of 1 being assigned to the low-inflation state. It is not surprising that it takes longer to gain credibility in a stochastic environment. Nevertheless, initial progress is

19. We verified, by repeating the exercise with different shocks, that the result is not a quirk.

relatively rapid. After 8 years in the stochastic simulations, the probability of State 1 is, on average, about 0.5. Credibility continues to rise at a fairly rapid pace for another 8 years, reaching 0.8. Progress becomes much slower after that, but credibility does continue to rise slowly with occasional minor setbacks. However, it settles at just over 90 per cent. After 50 years, there are still trials for which the policy regime has not become fully credible and a few for which the credibility index remains very low.[20]

The same panel shows the probabilities assigned to the other two states. Initially, the 3 per cent regime is given serious consideration. There is a brief shift to somewhat higher weight on the unit-root world as a major break begins in 1996 when agents become aware that the model of the 1960s and the 1980s is not performing very well. The unit-root model acts as a transition state, a place to stand in times of confusion when agents are unsure what the underlying inflation rate really is. This confusion does not last long, however, and the unit-root probability soon resumes its downward trend.

The top panel shows that, on average, inflation stays below the long-run target for a long time, although the differences are small after 5 years. The expectations of the uninformed agents, the Markov agents, come down sharply in the first few years, but then rebound and tend to stay just above the long-term target rate for most of the simulation. As in the deterministic simulations, we see relatively long periods of systematic forecasting errors based on continued weight being placed on the historical regimes.

We have not shown the average expectations, that is, the expectations we get when we combine the informed and uninformed agents. The informed agents have expectations closer to the actual outcome (though not the same, since they cannot know the shocks either). The average expectations approach 1 per cent per annum by about the 10th year and stay relatively close to 1 throughout the rest of the simulation, between the actual outcome and the Markov expectations.

20. Recall that this result is for the uninformed agents who use the Markov model. The model also includes the informed agents who know that they are in a low-inflation regime. Thus, the full measure of credibility ends at just over 93 per cent.

The third panel shows the output gap and the policy instrument. Because of the stickiness of expectations, achievement of the transition to the stochastic equilibrium takes quite some time. After a brief burst of activity in the recovery from the deep excess supply of the early 1990s, the output gap remains slightly negative, on average, through most of the simulation, and the policy instrument is, on average, held slightly above the steady-state level.

6.3 Variability across the trials

It is important to understand that the above results reflect an average across 60 trials with rather diverse individual results. In this section, we want to give a flavour for the variability around the average results. We begin with an indication of the overall variability by reporting five variables – inflation, the policy instrument, the output gap (Figure 12) and the credibility index and the Markov expectations (Figure 13) – each with their means and other indicators of the variation across the trials. Except for the credibility index, we show the mean and two sets of quantiles: 10 and 90 per cent, and 30 and 70 per cent. For example, for the line labelled 90 per cent, 90 per cent of the values are less than or equal to the value shown. To be clear, these quantiles are the descriptive statistics computed from the outcomes of the 60 trials at each time point. For the credibility index, since the distribution is far from normal, we add a number of other quantiles.

There are two points concerning variability of the economic variables. First, there *is* variability in the outcomes consistent with the maintained hypotheses of the experiment. The economic variables are roughly normally distributed, with dispersion greater than the dispersion in the shocks themselves, reflecting the promulgating mechanisms in the model and the imprecision of the controller. Inflation can be controlled to hit the target on average, but there is still considerable variation in the outcomes. The standard deviation for inflation settles at about 1 percentage point, so about 36 per cent of the time inflation is outside bands of +/- 1 percentage point around the target. However, with a high level of credibility, the consequences become smaller. One can see this in the graphs for the output gap and the policy instrument. As credibility reaches its peak, the variability of output and the policy instrument shrinks. For output, the standard deviation falls from about 1.33 in the transition period to about 0.9 percentage points at the end.

The reason for this is clear in what happens to credibility and the variability of expectations, as shown in Figure 13. Although the actual variability of inflation stays about the same, as credibility develops, expectations become less sensitive to the noise in the data and this reduces the overall volatility of the results. We think that this is a striking and very important result. We explore this issue further in considering the effects of shocks in a stochastic environment.

The picture for the credibility index is, of course, quite different. We can see clearly the strong skewness in the results. Ten per cent of the trials produce relatively rapid gains in credibility – much like the deterministic results. The median line shows that 50 per cent of the trials generate 100 per cent credibility by the 20th year, and 85 per cent of the trials have perfect credibility by the end. However, in 10 per cent of the trials, the true regime still has low credibility, even after 50 years.

6.4 Some interesting individual trials

6.4.1 Trial 37

The first individual trial is one where the shocks are moderate in the first few periods and credibility is established relatively quickly and never lost. Our Trial 37 (Figure 14) is such a case. Credibility jumps to 80 per cent in the first few years and full credibility is achieved over 7 years. Note that there is considerable volatility in observed inflation after full credibility has been established. Inflation exceeds 2 per cent or falls below zero fairly often, without shaking the credibility of the regime.

6.4.2 Trial 45

The next example is a trial where full credibility is eventually gained and retained, but it takes quite a long time. Our Trial 45 (Figure 15) provides an example. In this case, the difficulty comes from systematically negative shocks over the first part of the simulation, with one big positive cycle around 2010. Credibility is eventually established, but the process is long delayed and takes about 30 years to complete. In the first 20 years, the confused agents generally place a substantial weight on the unit-root state, except during the one period of higher inflation, when they jump to the 3 per cent view.

6.4.3 Trial 60

Next, we report a real "trial" for the unfortunate monetary authority. We have several very nasty sets of shocks in our drawings. One such case is Trial 60 (Figure 16). In this case, close to full credibility is attained by 2005, but a burst of inflation coming from both shocks, and what turns out to be unfortunate easing of the monetary instrument in response to the weak economy just preceding this episode, wipes out the gains. In this case, the uninformed agents quickly raise the probability assigned to the 3 per cent state. They do not return to the unit-root characterization; the weight assigned to it rises very slightly, but remains low throughout.

There follows a sequence of events that leaves agents oscillating back and forth between faith in the low-inflation regime and a return to the 3 per cent view. It takes another 20 years for the truth to begin to come through with lasting power. This does happen eventually, however, and credibility is then able to withstand a quite substantial combination of shocks in the final part of the simulation.

6.4.4 Trial 57

Next we consider a rather wild ride – our Trial 57 (Figure 17). In this case, despite some early buffeting, policy succeeds in gaining a high level of credibility. Then some very nasty surprises arrive. Credibility is lost completely in short order and, despite years of effort, never regained. Indeed, there is a trend downward in the credibility index. In this case, there is what appears to be increasing amplitude in the oscillation of inflation. This may indicate a case where our ad hoc policy rule is not adequate and that a better result could be found. On the other hand, everything we tried to modify the monetary control rule in order to produce a better result in such problem cases, had negative consequences for other trials. It appears that we cannot come to clear conclusions on this point without an optimal control algorithm.

We have not been able to identify any one particular feature of the trials that leads to sustained problems for the monetary authority. Obviously, there can be bad shocks that cause problems for a time. But we have persistent failure in some cases. We have looked at a number of indicators, including measures of variability, simple autocorrelation, cumulative runs and measures of covariation in the particular shocks. No single explanation can be offered as to why credibility is not attained in some cases. It is important to note that it is not just what happens to inflation that matters. However, we can say that the outcome for inflation matters a lot more when credibility has not been established.

It is difficult to know what to make of the problem trials. We must expect bad shocks from time to time and even sequences of bad shocks. Nevertheless, because we have some evidence that individual "fixes" can be found for individual problem cases, we have some reason to think that we can make further progress using optimal control techniques rather than an ad hoc rule.

6.5 Shocks in a stochastic environment

We have stressed that we view the stochastic environment as the appropriate one for policy analysis. In this subsection, we report briefly on experiments that mimic the analysis of the effect of credibility on expectations and nominal dynamics that we described for the deterministic environment. It is not as clear how to pose the question in a stochastic environment. At any point of notional calendar time in the future, there is a distribution of possible outcomes for credibility and the economic variables. For given credibility, the results of a shock will depend on the conditions of the economic variables. As well, across the trials there is considerable variability in the degree of credibility at any point in time.

Since we wanted to focus on the role of credibility in conditioning dynamics, we have proceeded as follows. We selected a single trial that yielded roughly average results. We use this trial to provide our control solution. We then introduce the same shocks we considered in the deterministic experiments, that is, one-period shocks occurring when there is partial credibility and when there is close to full credibility. The difference is that we now randomize the future following the shock and construct the distribution of shock-control paths from a fixed starting point. We report only the results for a 2 standard error shock to inflation. Inflation is close to the target when the shocks are experienced in both cases. Since the path of credibility differs in the control here from the path in the deterministic experiments, we introduce the shocks when conditions are similar as opposed to fixed points in calendar time. Thus, the first shock is introduced in 1999, when credibility in the control is about 0.6, while the second shock is introduced in 2008, when there is full credibility.

The results are summarized in Figure 18, which shows shock minus control values plotted against time from the shock. In the top panel, we show the average responses of the probabilities assigned to States 1 and 2. In the bottom panel, we show the policy instrument and the output gap.

When the shock arrives before full credibility has been established, we get results quite similar to those reported for the deterministic environment on impact and in the first few years. Credibility is virtually eliminated by the shock and remains low for another year before beginning to rise again. However, whereas in the deterministic environment the effects of the shock are largely gone by Year 5, in the stochastic environment, the shock has noteworthy, long-lasting effects. Even though the economic variables, including the expectations themselves, are essentially back to control after 10 years, the way expectations are formed is changed in a more lasting manner. Eventually, however, full credibility is re-established. It is important to remember that we are discussing the average results across 60 trials here. In many cases, the shock does not have such long-lasting effects, but in others the shock is reinforced by the subsequent random draws and there is a much more substantial effect on expectations formation.

The effects of the same shock when there is full credibility are dramatically different. They are so small they are difficult to see on the graph. Indeed, they are smaller than the effects of the same shock in a deterministic environment. The third set of lines is for the response to two consecutive 2 standard error shocks to inflation, introduced when there is full credibility. This has a more substantial, but still relatively modest effect on credibility and the economy. It seems that there is very strong non-linearity in the extent to which credibility can withstand shocks. This result has very important implications for policy. If these results are at all correct, there are large potential pay-offs in terms of reducing the volatility of cyclical fluctuations from establishing a stable, credible nominal anchor.

We hasten to note that we have more work to do before we can claim to have established these results as robust within our own framework. There are two aspects of our implementation that may be limiting the effects of shocks when there is full credibility. One is the restriction we imposed on the transition probabilities that make the unit-root state a transition state. In our characterization, there is no weight assigned to a direct jump from State 1 (1 per cent inflation) to State 2 (3 per cent inflation). There is a notional transition through State 3 (unit root). While this can happen simultaneously, and while we have shown several examples of simulations with sharp shifts from State 1 to State 2, we need to test the sensitivity of these results to the restrictions imposed on the MSM. Second, there is an issue with respect to how the autoregressive parts of the model are interpreted in a transition between states. With a more flexible formulation that treated the autoregressive structures differently when there was a transition between states than when a state was notionally continuing, we might get greater sensitivity to shocks. Nevertheless, the fact that two consecutive large inflationary shocks do not put much of a dent in credibility indicates that our result is unlikely to be changed qualitatively by any such extensions of the model.

6.6 Some sensitivity analysis

There are many questions one could pose and we have only a preliminary understanding of how this combination of Markov learning and macro simulation works. For this paper, we have completed two experiments to investigate the sensitivity of our results for the profile of the credibility index in the transition to a low-inflation environment. We look first at the sensitivity of the results to a change in the stochastic specification. We then look at a change to the calibration of the policy rule in which we raise the weight on interest smoothing. We report only the credibility index itself; the values are, as usual, the average across 60 trials.

Figure 19 shows the effects of changing the variability of the shock to inflation. Three lines are shown: the base case, where the standard deviation of the shock is 0.9; and two alternatives, one with the standard deviation reduced to 0.7, and the other with it raised to 1.0. We see that the degree of noise in the inflation process matters. Credibility comes much faster if there is less noise and hence a clearer signal that the regime has changed. Conversely, if the process is noisier, then the process is slower. Lower noise variance does not increase substantially the upper limit attained for average credibility, but the higher noise variance does have a more significant effect in the other direction. Nevertheless, there is still clear progress towards building credibility in the new regime.

The final evidence we consider is what happens when we raise the weight on instrument smoothing in the policy rule. We raise the coefficient on the lagged instrument value from 0.1 to 0.25, lowering the weight on the target function correspondingly. The results are shown in Figure 20. The main point is that this change has little effect on the results. However, it does delay slightly the process of gaining credibility. With less attention to keeping inflation close to the target, the signals are more confused and it takes uninformed agents longer to learn that there has been a change of regime. Once credibility has been established, however, the greater emphasis on instrument smoothing seems to work marginally better. This suggests that the optimal policy rule depends on the state of credibility, which we find quite plausible, and that our particular ad hoc rule may have a bit too much response to shocks for a world with credibility.

7. Conclusions

In this paper, we have described a representation of how uninformed agents, in other words, agents who use only information on inflation in making judgments about the policy regime, form their expectations. The model allows for a small set of explicit regimes to be entertained as possible states at each point in time, with agents reacting to the data in assigning probabilities to each regime. Expectations are then generated consistent with these judgments. Learning takes place through time in the sense that there is automatic updating of the probabilities in the light of new experience.

We reported estimates of a particular version of this framework for the 1954-92 period. The model has three states: low inflation (1 per cent per annum); inflation at roughly the historical mean (3 per cent per annum); and a unit-root state, which has no formal anchor in the sense of a mean inflation rate. This state is used, historically, to describe parts of the periods of high inflation. It also serves as a transition state at other times, acting as a kind of rule-of-thumb (inflation is going to remain about where it is) in times of change. The states with nominal anchors, where there are long-term means, are estimated to have persistence properties. Inflation and short-term expected inflation need not be close to the presumed long-term means.

With this characterization of expectations formation, one can observe relatively long periods of systematic errors in expectations, especially when there are large shocks or changes in the policy regime. We see this in both the historical estimates and the properties of the expectations in our simulations.

Representing expectations in this way has several major advantages. It can help explain some historical puzzles, such as the behaviour of interest rates in the 1970s. Moreover, we have shown that the measures are consistent with the available survey data on inflation forecasts, which contain systematic errors from an *ex post* perspective. More importantly, this model permits us to understand the importance of the difference between point forecasts and the underlying credibility in the inflation regime. Two different probabilistic representations can give similar one-period-ahead forecasts, but contain quite different longer-term forecasts. This point is very important for policy analysis because the policy initiatives necessary to counter shocks depend much more on the underlying characterization of the regime by agents than on the particular point expectations at a moment in time. We illustrated this fact with simulations of the same shock under two different conditions with respect to credibility.

The Markov model also allows us to study the learning process. We did this, to some extent, in our review of the historical estimates. However, our main effort in this direction was contained in simulations of the future based on the starting conditions in Canada in 1993. For these simulations, we specified and calibrated a simple model of the economy and the policy control process. The model was simulated interactively with the model of expectations formation. In this part of our work, we introduced a distinction between informed and uninformed agents. The Markov model uses only information about inflation in the formation of expectations. For our simulations, we assumed that there are also informed agents who form expectations based on a correct characterization of the economy and policy. These agents form model-consistent expectations.

We began with deterministic simulations to investigate what the initial conditions and the properties of the estimated learning process have to say about the process of gaining credibility for a low-inflation regime. We found that even without shocks, the process would take about 8 years. We used this base line to introduce one-time shocks to inflation at different points along the path. These simulations allowed us to show that the underlying model used by uninformed agents matters a lot to the dynamic properties of an economy experiencing a shock and to the nature of the policy control problem.

We also showed that credibility can be fragile and easily lost in the face of shocks, at least until agents develop a high level of confidence in the regime. We developed this idea with both deterministic and stochastic simulations of the model.

The model says that in 1992, the credibility of the low-inflation regime was just beginning to develop. Given our stochastic specification, it appears that the monetary authority can be confident that credibility in the low-inflation regime will rise substantially based on the demonstrated commitment to it. However, this can be expected to entail a relatively long period of moderate tightness to keep inflation from escalating by chance such that agents jump back to the old model. There could be a fortunate set of shocks for which the path would be easier. However, there could also be an ill-fated set of shocks for which attaining full credibility becomes virtually impossible.

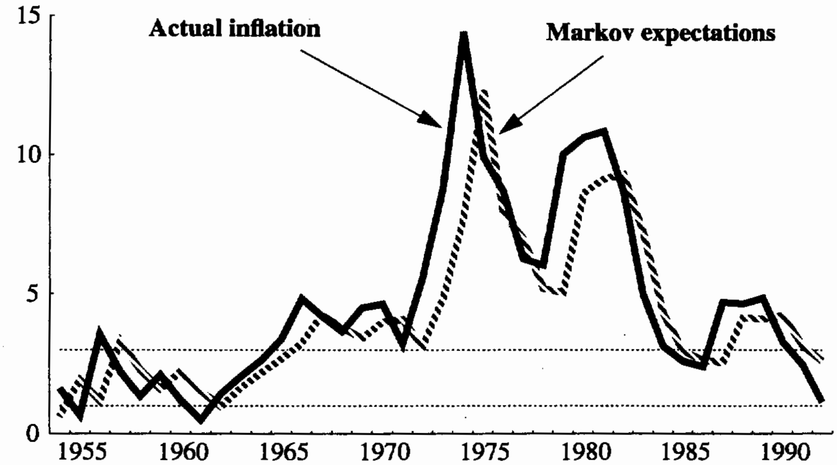
The shock analysis provided striking predictions about possible gains in terms of reduced volatility in the economy if full credibility can be achieved.

Appendix

Figure 1

Markov Switching Model for Canada

Actual and Markov-Expected Inflation
(GDP deflator)



Probabilities of States 1, 2 and 3

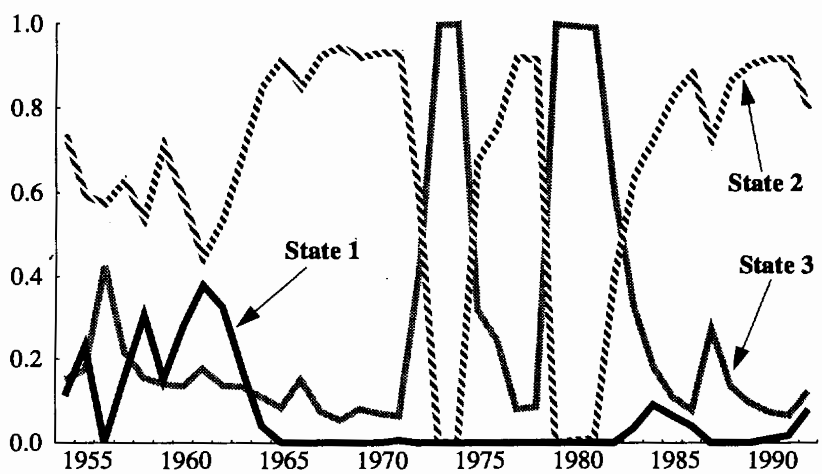


Table 1

Maximum-Likelihood Estimates of the 3-State Markov Model

Parameter	Estimates	Standard error	P-value
γ_1	0.7	imposed	
ρ_1	0.3	imposed	
γ_2	1.10	0.30	0.0001
ρ_2	0.64	0.06	0.0000
γ_3	0	imposed	
ρ_3	1.0	imposed	
α_1 *	0.18	0.48	0.3539
α_2 *	1.26	0.34	0.0001
α_3 *	0.37	0.38	0.1630
σ_1	0.70	0.36	0.0201
σ_2	1.02	0.15	0.0000
σ_3	2.74	0.41	0.0000

*** The implied estimates of p, q and r are: $p = 0.57$, $q = 0.90$ and $r = 0.65$.**

Figure 2

Inflation and Alternative Measures of Expectations

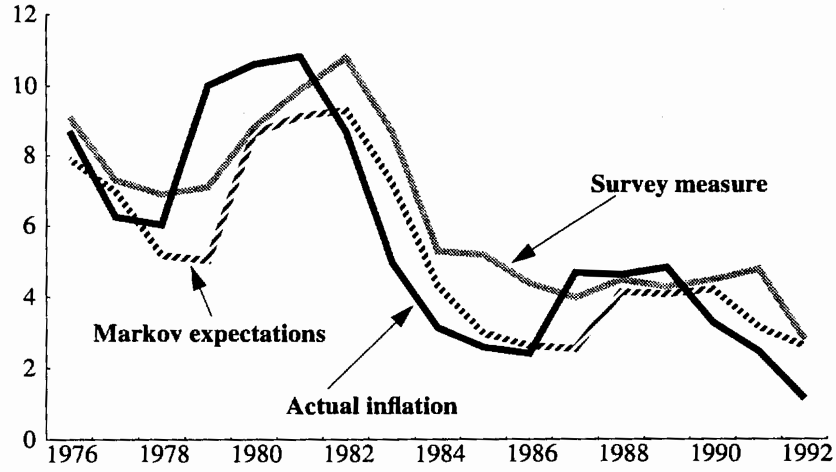


Figure 3
Deterministic Simulation: Base Case

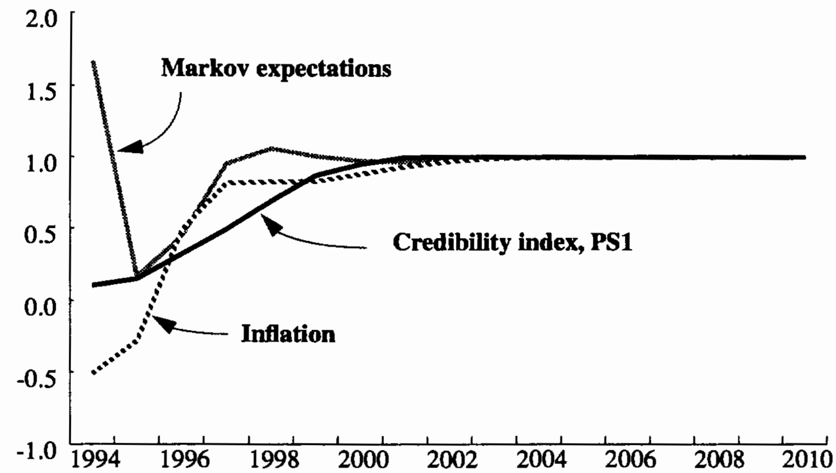


Figure 4
Shocks to Inflation Under Partial Credibility: Credibility Index

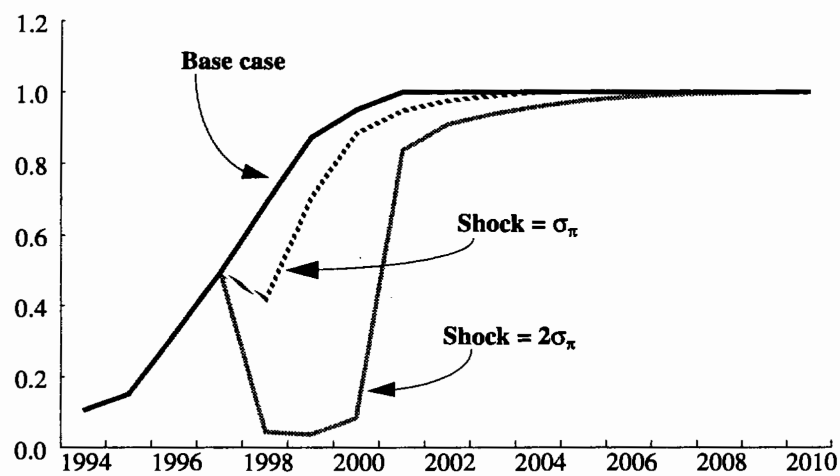


Figure 5

Shocks to Inflation Under Partial Credibility: Policy Instrument

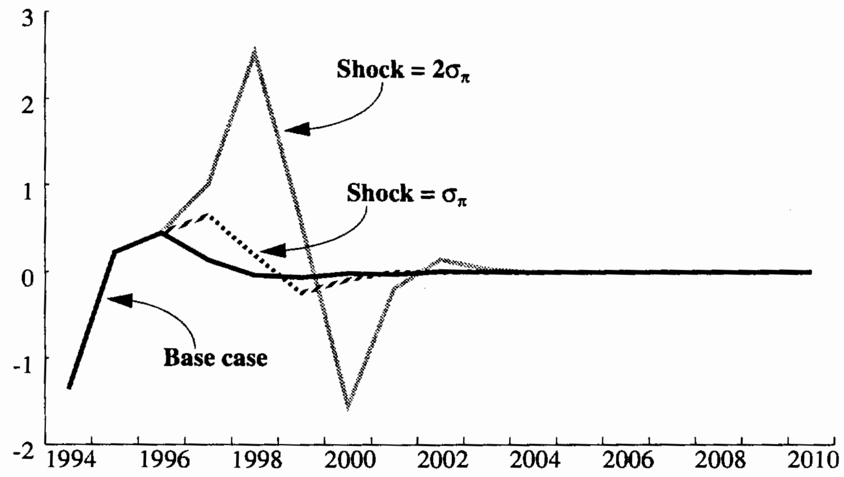


Figure 6

Shocks to Inflation Under Partial Credibility: Output Gap

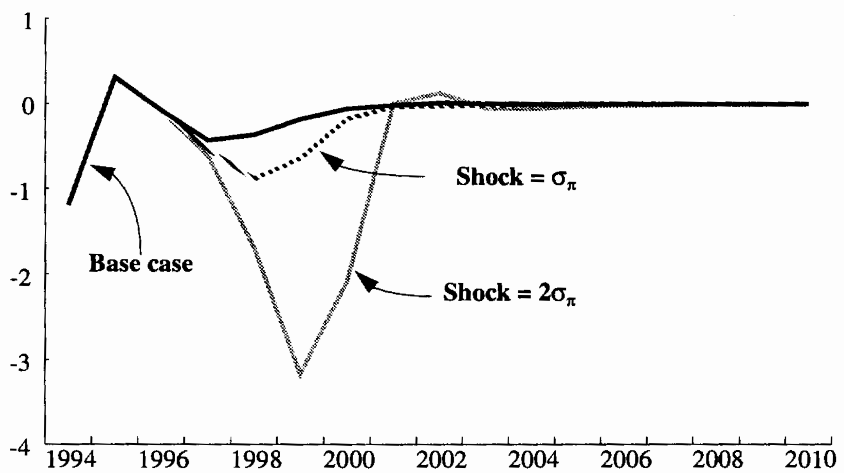


Figure 7

Shocks to Inflation Under Differing Credibility: Credibility Index

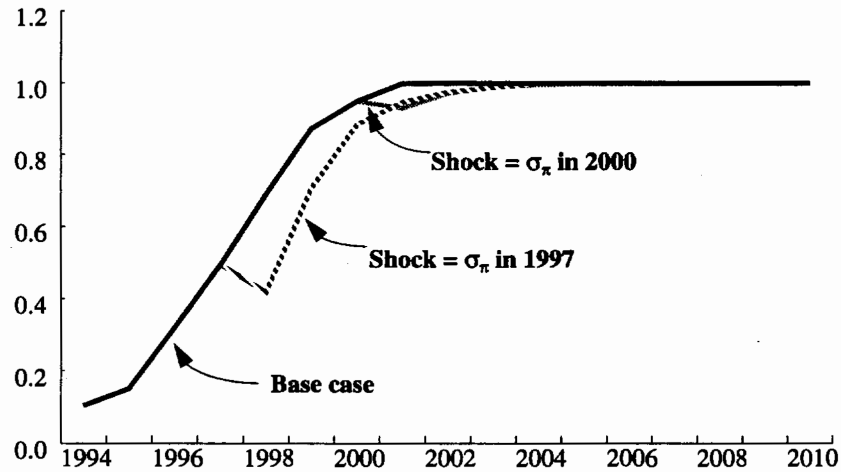


Figure 8

Shocks to Inflation Under Differing Credibility: Output Gap

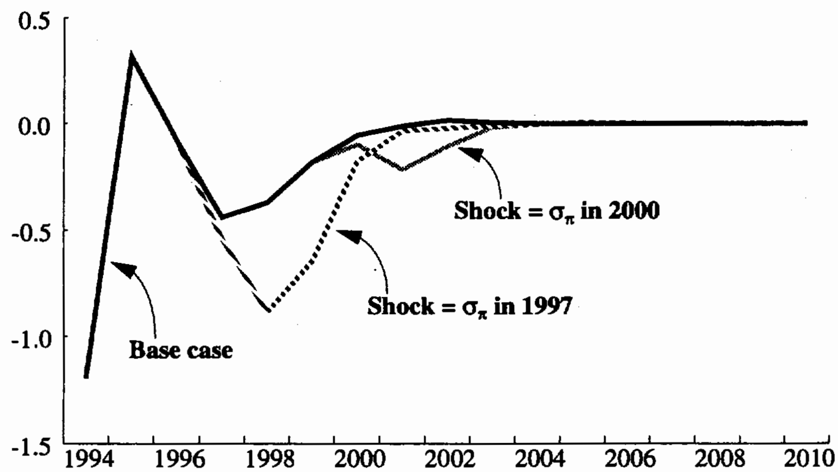


Figure 9

Larger Shock to Inflation Under Differing Credibility: Credibility Index

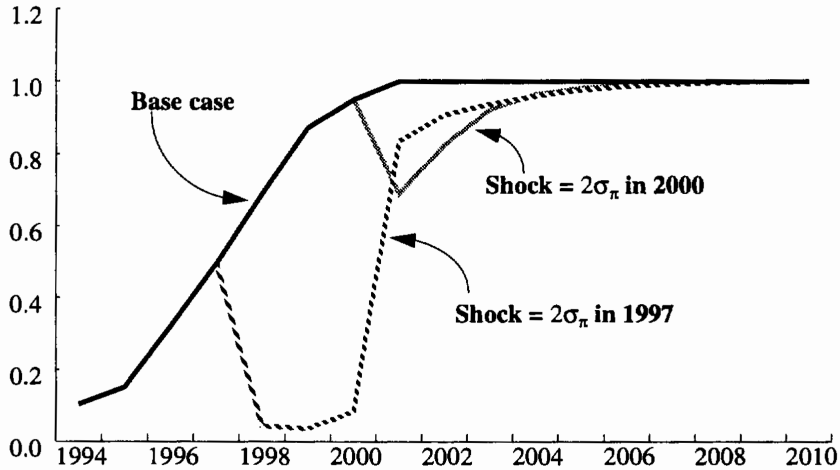


Figure 10

Shocks to Inflation Under Full Credibility

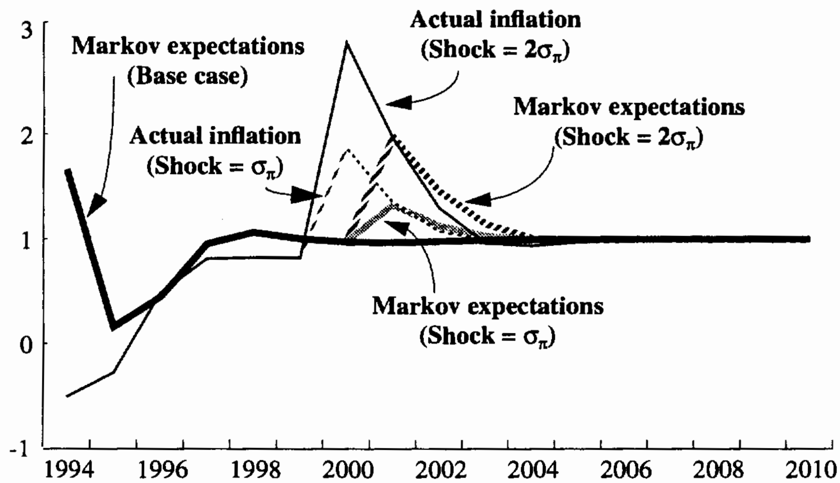


Figure 11

Results of Stochastic Simulations: Average Over 60 Trials

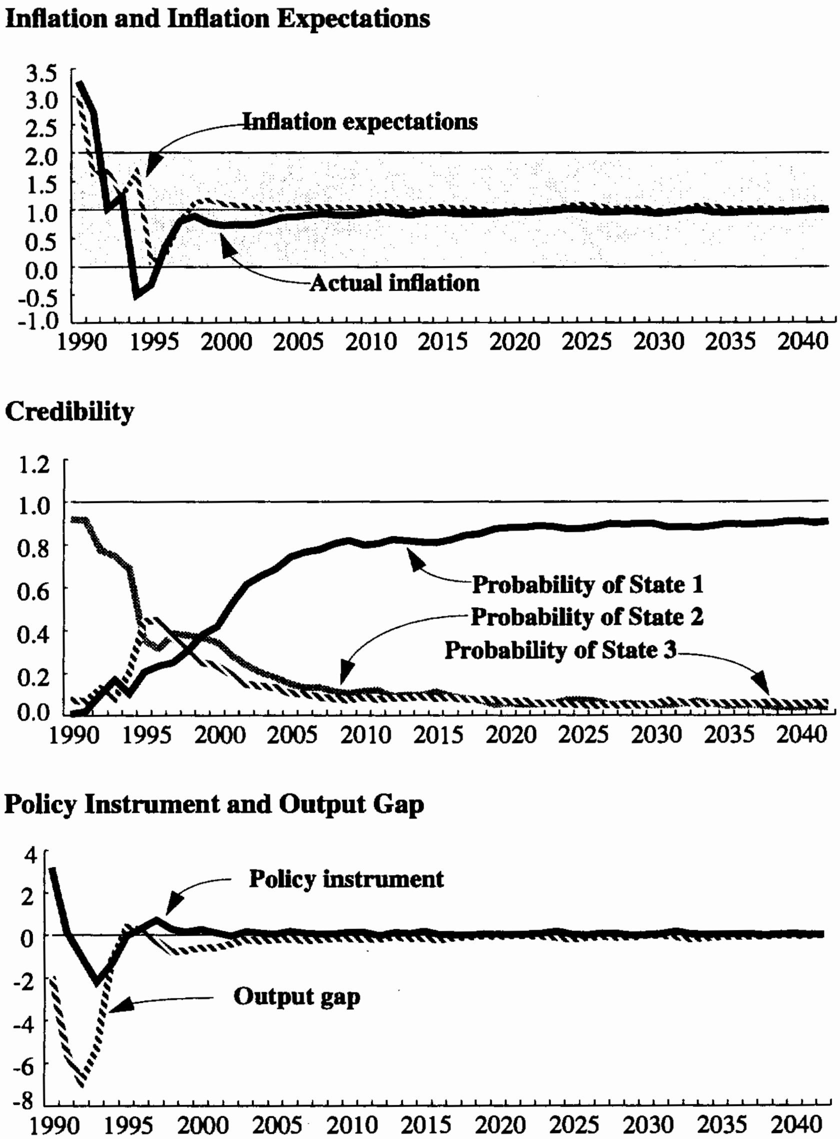
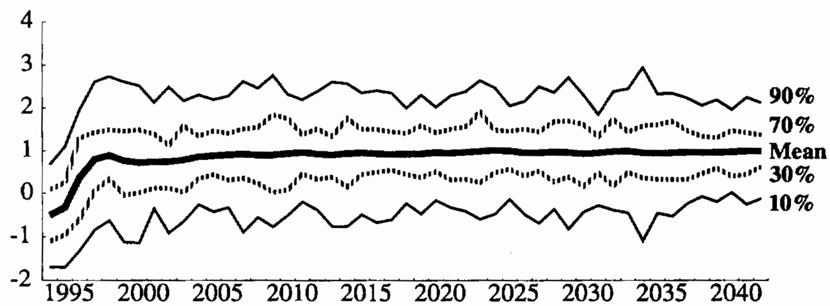


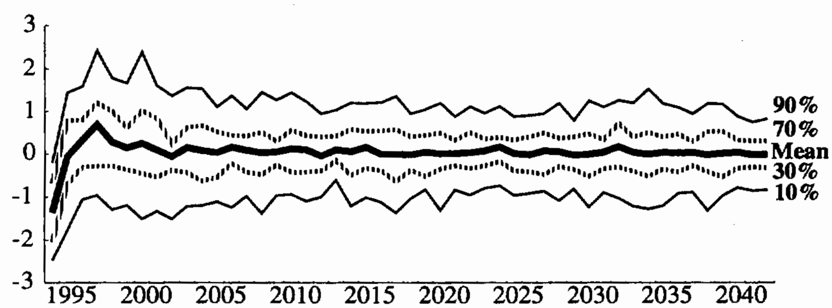
Figure 12

Distribution Across the Trials as Indicated by Various Quantiles

Actual Inflation



Policy Instrument



Output Gap

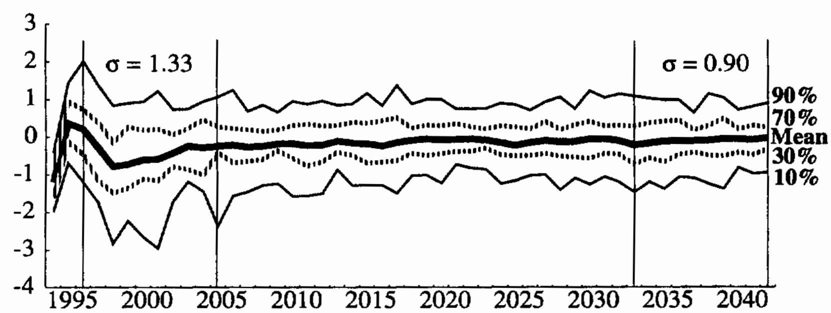
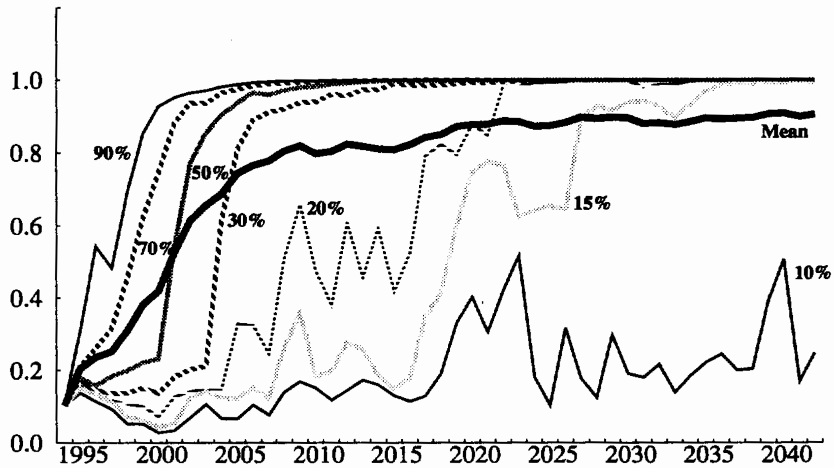


Figure 13

Distribution Across the Trials as Indicated by Various Quantiles

Credibility Index



Markov Inflation Expectations

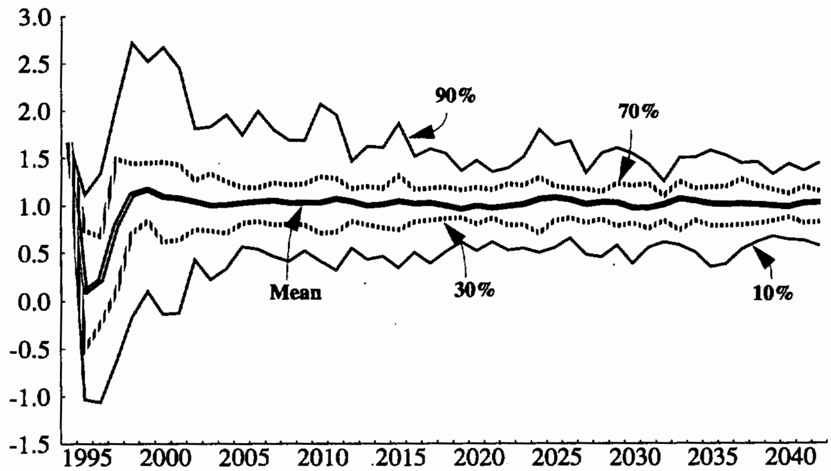


Figure 14
Trial 37

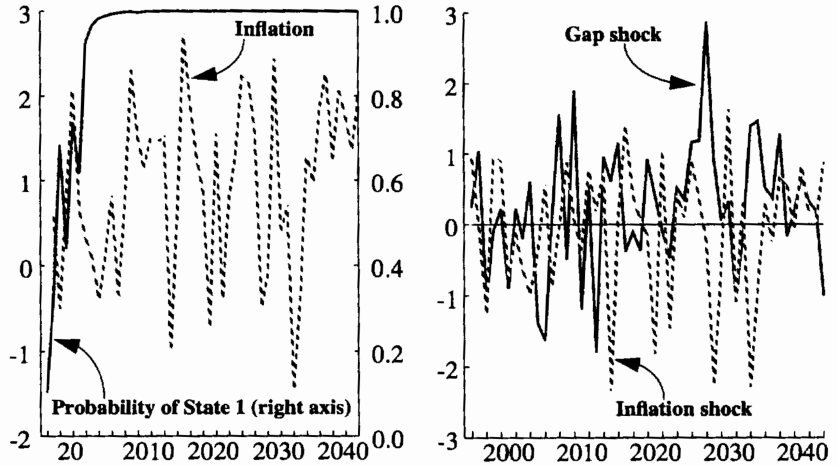


Figure 15
Trial 45

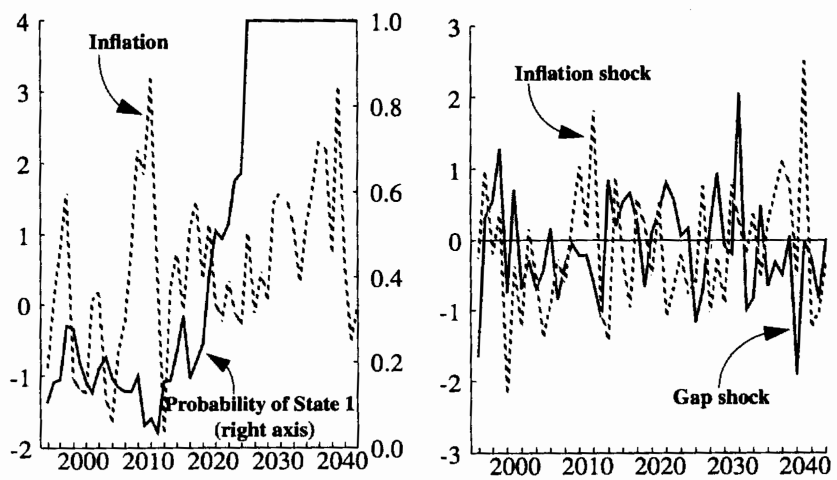


Figure 16
Trial 60

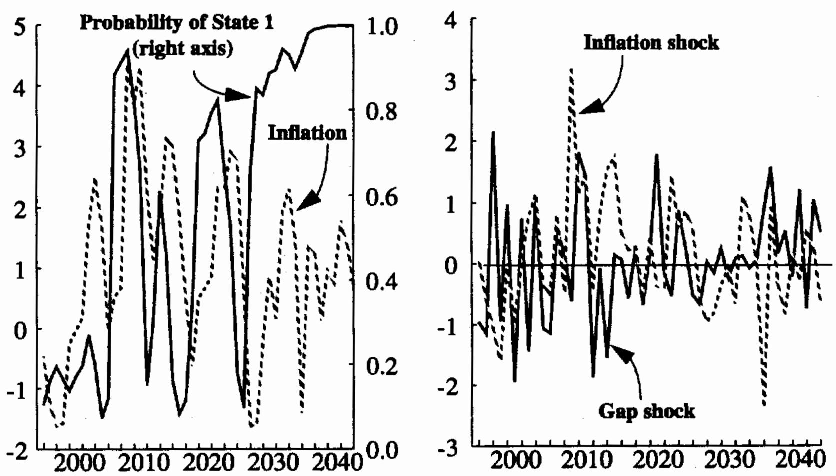


Figure 17
Trial 57

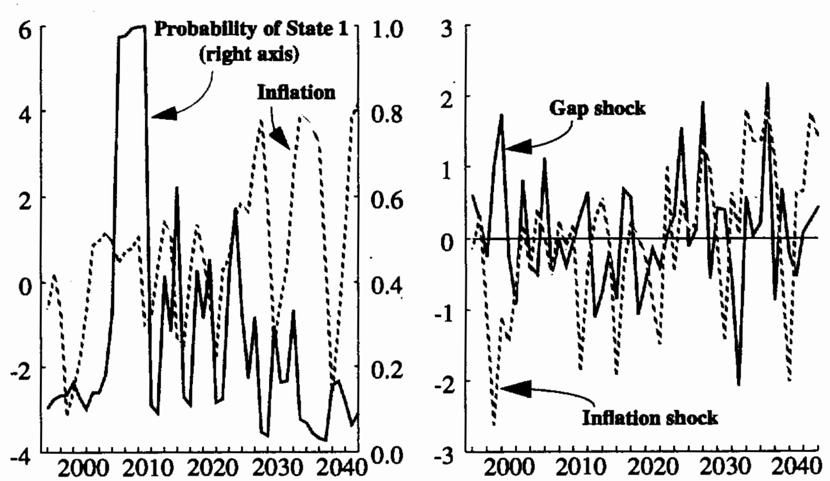


Figure 18

Stochastic Simulations: Shocks to Inflation Under Differing Credibility
(Shock – control)

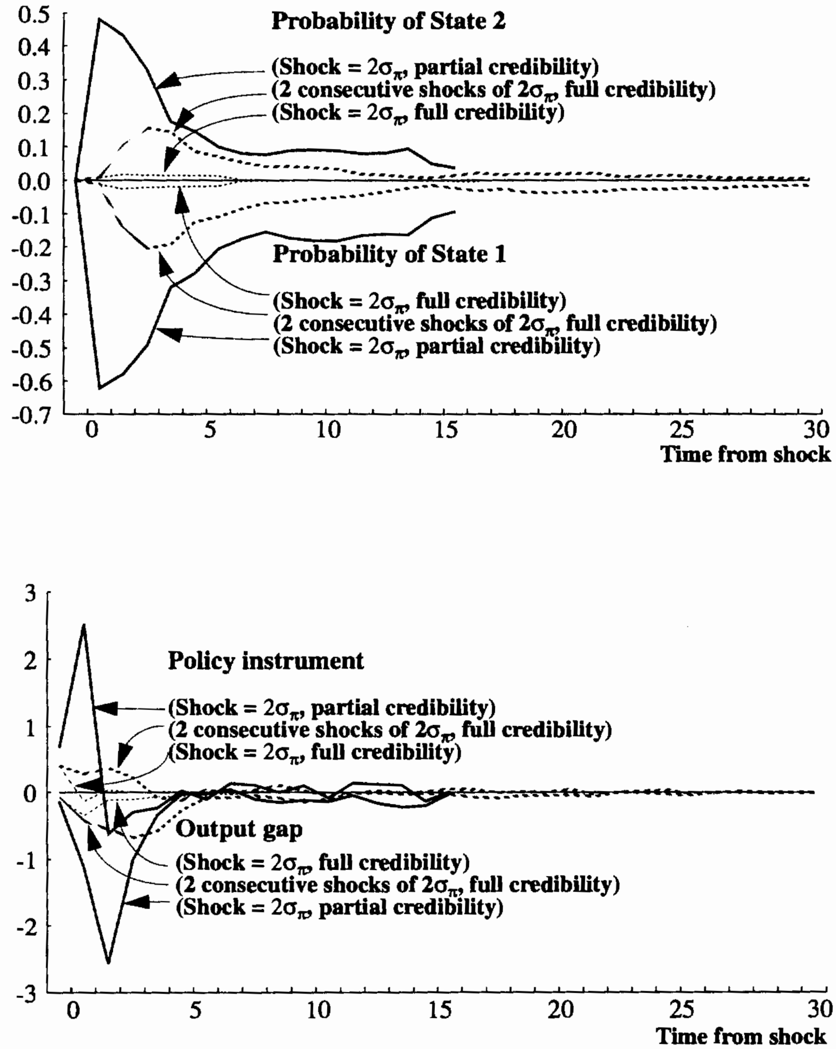


Figure 19

The Influence of σ_π on the Credibility Index

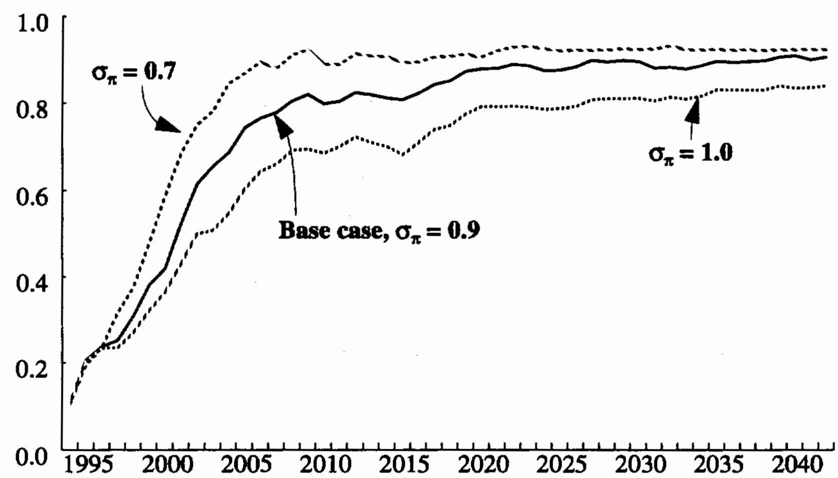
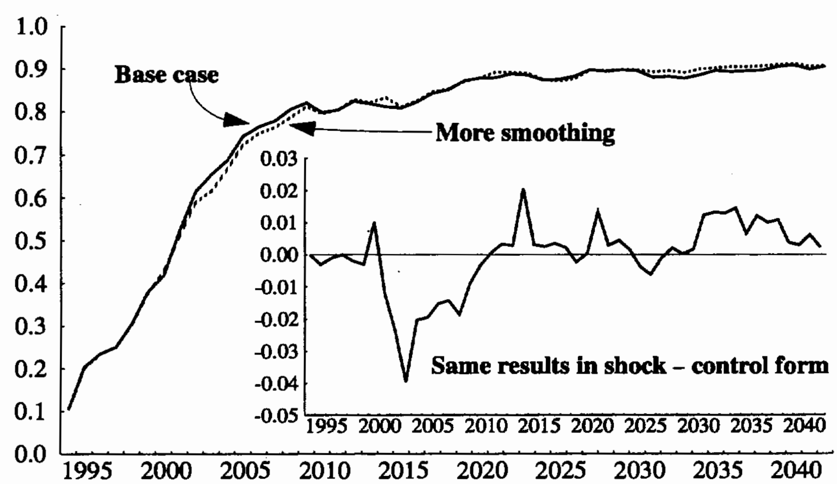


Figure 20

The Influence of Instrument Smoothing on the Credibility Index



Discussion

*Louis Phaneuf**

Introduction

Laxton, Ricketts and Rose present a macroeconomic model which, by assumption, characterizes the past performance of inflation in terms of three possible states of the world (or inflation regimes). In their model, a certain percentage of individuals (30 per cent) form rational expectations in the sense that they know what model is being used by the monetary authority. The remaining individuals (70 per cent) do not know the model used by the monetary authority and, instead, form their expectations using an information-learning mechanism based on a Markov switching model.

The authors begin by estimating the Markov expectations-formation model using Canadian data for the 1954-92 period. According to the estimation, there were relatively long periods during which systematic errors in inflation expectations occurred. The authors note the apparent connection between this result and the fact that inflation predictions based on survey data indicate that forecast errors are autocorrelated and systematically different from zero. The

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authors argue that this establishes the validity of a Markov expectations-formation mechanism and justifies its use in a simulated macroeconomic model.

Next, the authors present a macroeconomic model that incorporates the Markov expectations-formation mechanism and a monetary policy rule. The model is first standardized or calibrated, then simulated with the assumption that the monetary authority subscribes indefinitely to a regime aimed at achieving an average long-run rate of inflation of 1 per cent. The purpose of this simulation is to gauge in both a deterministic and a stochastic environment how well the monetary authority can establish the credibility of this regime and how long it might take to do so. The authors also evaluate the percentage decline in output relative to potential that would be needed to restore the credibility of a low-inflation regime should the economy be affected by inflationary shocks in the future.

My comments on the text are organized as follows. I begin by examining the question of inflation regimes and expectations formation. Next, I address a question that might seem curious: Is greater credibility always a good thing? I conclude my comments with some remarks on the strategic complementarity between informed and uninformed individuals.

Inflation Regimes and the Formation of Expectations

What is *theoretically* the main contribution of the literature on various inflation regime models? In my opinion, the salient point of these models is that, at least in principle, they offer *one* possible explanation for the systematic discrepancies observed between survey-based inflation predictions and actual inflation performance. Most often, such differences are attributed to a lack of rationality on the part of individuals, who apparently ignore useful items of information. The approach adopted by the present authors and others, is to consider that such systematic forecast errors arise because individuals do behave rationally but are faced with shifts in the inflation regime.

Let us start with a simple example that demonstrates how a Markov regime-switching model is related to the formation of inflation expectations. Assume that inflation is conditioned by two processes (or regimes) and that the shift from one regime to the other is controlled by an unobserved state variable ω_t, which can be either one or zero. If we define α_t as the probability that the rate of inflation over period π_t will be influenced by the State 1 process, given all information available to individuals at time t, (I_t), then we can express the anticipated inflation rate as follows:

$$E_t\pi_{t+1} \equiv E(\pi_{t+1}|I_t) = \alpha_t E(\pi_{t+1}|\omega_{t+1} = 1, I_t) + (1-\alpha_t) E(\pi_{t+1}|\omega_{t+1} = 0, I_t).$$

Thus, the *ex post* forecast error is as follows:

$$\pi_{t+1} - E(\pi_{t+1}|I_t) = [\pi_{t+1} - E(\pi_{t+1}|\omega_{t+1} = 1,\ I_t)] + (1-\alpha_t)[E(\pi_{t+1}|\omega_{t+1} = 1,\ I_t) - E(\pi_{t+1}|\omega_{t+1} = 0,\ I_t)].$$

Let us now assume that inflation is currently controlled by a State 1 process. The first term on the right in the above equation then has a null mean – i.e., the observed rate of inflation is not correlated with the conditional forecasts of the State 1 process. However, the second term is a different matter. Providing that individuals assign a certain probability to inflation obeying a State 1 process $((1-\alpha_t) > 0)$ in period $t+1$, and providing that conditional expectations at the two states are different, the mean of the second term on the right will not be null. The difference between the two expectations may thus be correlated with the current items of information contained in I_t. *Ex post* predictions will appear to be biased even though the available information was used optimally.

In practice, however, implementing this idea is problematic. In this paper, the authors estimate a Markov regime-switching model by assuming *a priori* that there are three possible states: "low inflation," where inflation is assumed to be governed by a stationary process producing an average long-run inflation rate of 1 per cent (State 1); "mean inflation," where inflation is assumed to be characterized by a stationary process with an unknown mean and unknown autoregressive structure (State 2); and "high inflation," where the inflation process is constrained to be non-stationary (State 3). "Uninformed" individuals form their inflation rate expectations using the Markov switching model with the constraint that the probability of direct transition between stationary regimes (low inflation and mean inflation) is null.

Characterizing the possible states is not an easy task. Studies using similar methodologies have been carried out on the U.S. economy (e.g., Garcia and Perron 1990; Brunner 1991; and Raymond and Rich 1992). According to these studies, over the 1955-92 period: i) inflation may have been characterized by several regimes; ii) regime shifts appear to have coincided with oil price shocks; and iii) when the estimation method is adjusted to capture possible structural breaks in the inflation process, inflationary shocks during the 1970s no longer necessarily appear to be permanent (i.e., the assumption of a non-stationary inflation state is violated). It is obviously easier to adhere to the chosen regime when the choice among regimes is based on precise estimation and rigorous tests. This leaves it to uninformed individuals to solve a problem on which even "informed econometricians" cannot agree.

The facts reveal that survey-based inflation predictions follow the observed rate of inflation with a lag. The Markov regime-switching model may well generate predictions that are consistent with the facts, but it is not the only model that can do so. A simple adaptive-expectations model can produce the same results. The adaptive-expectations hypothesis was neglected for several years, supposedly because it assumed a lack of rationality on the part of individuals. However, recent developments give reason to question this assumption. For example, Mankiw (1985), Akerlof and Yellen (1985), Blanchard and Kiyotaki (1987) and Ball and Romer (1990) have studied the microeconomic basis of nominal rigidities. They have demonstrated that when firms face price-adjustment costs, they may find it optimal *not* to change prices in response to nominal changes. Ball (1991) argues that the main results of research into "menu costs" may be applied to the problem of expectations formation. Firms will be prepared to accept the cost of "sophisticated" forecasts if the expected gains are high. These gains appear to be fairly low, however, because adaptive expectations are *almost* rational.

If the value of an expectations-formation mechanism is to be judged on the basis of how coherent its predictions are compared with predictions based on survey data, it would be interesting to impartially *test* the coherence between survey-based predictions and those generated by various expectations-formation mechanisms, including the adaptive-expectations mechanism. Evans and Wachtel (1993) propose just such a test.

Is Credibility Always a Good Thing?

The macroeconomic model used in the simulation is a simple aggregate demand/Phillips curve model. The monetary authority hopes to establish a high level of credibility in a regime that aims to deliver an average long-run inflation rate of 1 per cent. As is the case for many models used to study the link between credibility and disinflation (e.g., Taylor 1983; Ball 1990b and 1991), the aggregate demand side is very simple and does not directly incorporate inflation expectations. In a model where inflation expectations affect only individuals' wage demands (an "augmented" Phillips curve), greater credibility in a more restrictive monetary policy in a context of positive shocks to prices and wages is always beneficial; it promotes moderation of wage demands, which in turn mitigates the decline in aggregate output associated with unfavourable shocks.

However, this happy state of affairs may be complicated by a number of factors. McCallum (1988) studied the impact of greater credibility in a macroeconomic model where not only workers but also consumers base their current decisions on the expected rate of inflation. He shows that greater credibility in a more restrictive monetary policy leads to a sharper drop in permanent income in response to a positive price shock and thus to a larger decline in consumption. Meanwhile, Howitt (1991) points out that aggregate demand is positively dependent on the anticipated inflation rate as long as the interest elasticity of the demand for money is negative and total spending is dependent on the real interest rate. He notes that greater credibility in a disinflation policy may produce a stronger recessionary effect.

The lesson to be drawn from the foregoing discussion is that greater credibility is a double-edged sword in a macroeconomic model in which inflation rate predictions can directly influence individuals' current decisions, on both the aggregate demand and aggregate supply sides.

Strategic Complementarity Between Informed and Uninformed Individuals

I would like to conclude my comments with some remarks on the economic role of uninformed individuals. It might seem reasonable that the importance of this category of individuals for the economy would depend on their proportion. In this paper, uninformed individuals are in the majority, accounting for 70 per cent of the total population. However, high numbers are not necessarily required to play a significant role in the economy. Recent work on the strategic complementarity between individuals with a "sophisticated" approach to forming predictions (i.e., individuals forming rational expectations) and individuals with a more "naive" approach (i.e., individuals forming adaptive expectations) indicates that the importance of naive agents is disproportionately large. In other words, their influence on economic developments is much stronger than suggested by their relative weight in the economy (Haltiwanger and Waldman 1989). The explanation for this finding is that the existence of strategic complementarity encourages sophisticated agents to develop a rational incentive to imitate naive agents at equilibrium.

Session 3: Response and General Discussion

Summary by Irene Ip

David Rose noted that his discussant was right to question the assumptions on the Markov model since technical problems meant that the implementation in his and his co-authors' paper was not ideal, and that he too was worried about the restrictions used. However, he suggested that there was no need to overstate the implications of both of these problems for the analysis since the probability charts show that the movements from one state to another are virtually simultaneous. On the issue of the relative importance of regime uncertainty, he couldn't say for sure, but he believed that it would probably be necessary at least to go further than the fixed-parameter models to avoid the Lucas critique.

However, he argued that regimes and information-processing costs have to be taken seriously since the rational expectations ideas, while conceptually useful, do not work when economists try to model the real world, even in those models with rational decisions under constraints. Given that his and his co-authors' model converges to first-order rationality, although the second moments were not necessarily Muth-consistent, he asserted that this model made a useful contribution.

Conference participants agreed with Rose on this latter point. For example, Peter Howitt stated that he considered the work reported by Rose to be an "exciting project" even though, like all first efforts, there were a number of ad hoc elements. In particular, he saw the use of the unit-root state for transition as being associated with the problem of establishing credibility and wondered what would happen if the transition process were changed. Rose indicated that they have been studying the sensitivity of the Markov-model results to various assumptions and have often been surprised, so it would be difficult to say what would be the effect of a different hierarchy in the transition process.