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**Government Debt, Life-Cycle Income and Liquidity Constraints:
Beyond Approximate Ricardian Equivalence***

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Abstract

Evans (1991) has demonstrated that Blanchard's (1985) finite-horizon model obeys approximate Ricardian equivalence. We show that this result is determined largely by an unrealistic assumption that labor income grows monotonically over a consumer's entire lifetime. Introducing more realistic lifetime earnings profiles, we find that the effects of government debt on the real interest rate and the capital stock become considerably larger. In particular, leaving aside the effects of distortionary capital taxation, the extended model with liquidity constraints predicts that real interest rates would decline by about 150-200 basis points if government debt were eliminated completely in all OECD countries.

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SUMMARY

The effects of government debt on the real interest rate, or the degree to which government deficits crowd out private-sector economic activity, depend to a large extent on whether consumers view debt as net wealth. If consumers are connected to all future generations and can borrow and lend against their future income streams, changes in debt will not crowd out private consumption and investment because consumers effectively internalize the government's intertemporal budget constraint and regard a debt-financed reduction in taxes today as implying an equivalent increase in future tax burdens. This extreme polar case is generally referred to as the Ricardian equivalence hypothesis.

This paper uses an extended version of Blanchard's finite-horizon model to re-examine the validity of the Ricardian equivalence hypothesis. The model has been used extensively to study fiscal policy issues because it captures the essential intertemporal aspects and gives rise to well-defined steady-state properties. It has been demonstrated that although agents are disconnected from future generations in the model, the model obeys approximate Ricardian equivalence; debt financing has very small effects on the equilibrium real interest rate and capital stock.

As the paper shows, incorporating more realistic individual lifetime income profiles and liquidity constraints into the Blanchard model can significantly change estimates of the long-term implications of government debt. OECD estimates of aggregate net public debt for 18 of the largest industrial countries show an increase of about 20 percentage points in the debt-GDP ratio since the late 1970s. The model developed in this paper suggests that this increase in government debt has caused an increase in the world real interest rate of 76 basis points and a permanent reduction in world real GDP of 2.9 percent.

I. INTRODUCTION

The persistent accumulation of government debt is attracting an increasing amount of attention in policy debates. According to OECD estimates, net public debt as a percentage of nominal GDP for the G-7 countries increased from 21.1 percent in 1979 to 38.6 percent in 1993. Based on available data, it is clear that government debt has continued to grow rapidly in the G-7 since 1993. These trends have not been restricted to the major industrialized countries.¹ Figure 1 plots an aggregate measure of net public debt that is based upon 18 OECD countries, and separately for Belgium, Canada, Italy, and the United States.² Many analyses examining the implications of higher public debt have focused on a particular country and, consequently assume that there will be no impact on the equilibrium world real interest rate. Although this assumption may be appropriate when a country is relatively small, it may not be appropriate for a large country like the United States. Furthermore, although some of these countries individually may cause a negligible effect on the equilibrium world real interest rate, the fact that so many of these countries together have run persistent government deficits imply that the combined effects of the debt buildup may have been large.

Barro (1974) has shown that the effects of government debt on the real interest rate, or the degree of crowding out, depends to a large extent on whether consumers view debt as net wealth. If consumers are connected to all future generations and can borrow and lend against their future income streams, changes in debt will not crowd out private consumption and investment because consumers effectively internalize the government's intertemporal budget constraint. In such a world, agents effectively offset the effects of current taxation in order to meet their future tax liabilities. This invariance proposition is commonly referred to as "Ricardian equivalence," although David Ricardo himself did not believe that tax financing and deficit financing were equivalent. Indeed, Ricardo preferred financing current expenditures with taxes precisely because he believed that if government expenditures were financed by issuing debt, the private sector would underestimate their future tax liabilities, and, in such circumstances, would overconsume currently available resources.³

In this paper, we use an extended version of Olivier Blanchard's (1985) finite-horizon model to re-examine this issue. This model has been used extensively to study fiscal policy issues because it captures the essential intertemporal aspects and gives rise to well-defined steady-state properties that are absent from ad hoc IS-LM models. Evans (1991) has demonstrated that despite the fact that agents are disconnected from future generations in the Blanchard model, the model obeys approximate Ricardian equivalence; debt financing has very

¹See Alesina and Perotti (1995) for a survey of explanations for this buildup in debt.

²In addition to the Group of Seven Countries, this aggregation includes Austria, Australia, Belgium, Denmark, Finland, Greece, Ireland, The Netherlands, Norway, Spain, and Sweden.

³See O'Driscoll (1977) and Sraffa (1951) for a discussion of Ricardo's views on this question.

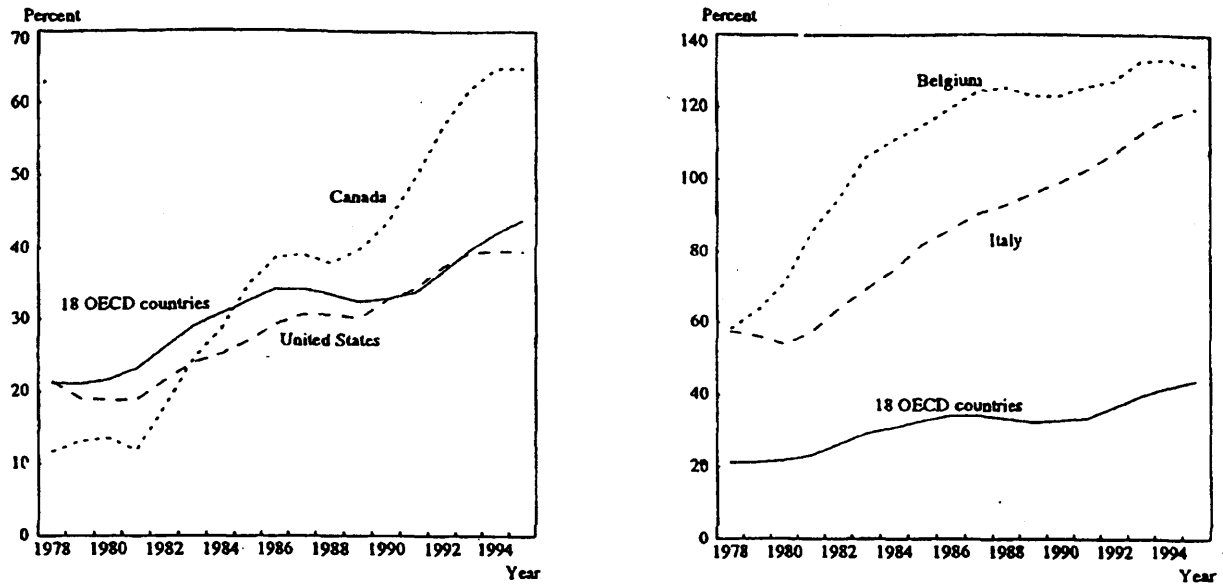


Fig. 1. Measures of Net Public Debt as a Percent of GDP

Source: OECD Economic Outlook, June 1994

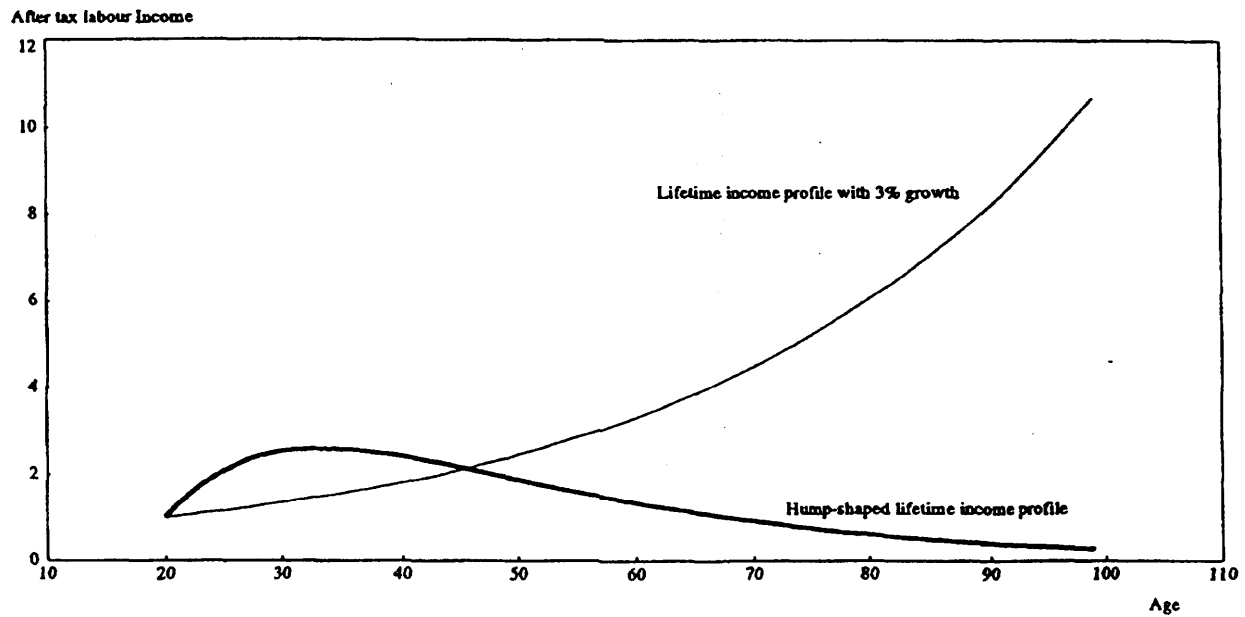


Fig. 2. Life-Time Income Profiles

(Income is normalized to 1 at age 20)

The hump-shaped profile assumes 1.5% aggregate productivity growth (see equation 10 on page 15)

small effects on the equilibrium real interest rate and capital stock. However, by assuming that an agent's income grows monotonically over time, Evans effectively treats agents as dynastic households (rather than individuals) with finite horizons. Not surprisingly, under this interpretation the deviations from Barro (1974) are very small. Alternatively, one could interpret the Blanchard model from a life-cycle perspective, consisting of overlapping generations rather than dynasties.⁴ In this paper, we extend the Blanchard model to allow for realistic individual lifetime-income profiles and to incorporate the finding that consumers in the early part of their life cycle cannot borrow against their future income. We show that these life-cycle modifications are sufficient to generate significant crowding-out effects from government debt in the spirit of Diamond (1965).

The remainder of this paper is organized in the following way. Section II extends the basic Blanchard model to allow for more realistic income profiles and the possibility that individuals cannot borrow against their future income stream in the early part of the life cycle. Section II embeds the structure of the extended model into a quantitative closed-economy model in order to estimate the long-run effects of world government debt. Section III has two objectives. First, we show why Ricardian equivalence has to hold up to a very close approximation in the original Blanchard model. Second, we show that a calibration of the model with more realistic life-cycle considerations produces significant increases in real interest rates and large ongoing crowding-out effects from government debt. In Section III, we report some simple reduced-form empirical evidence that suggests real interest rates in countries with liberalized capital markets have been affected by the buildup in world government debt. Section IV concludes by discussing some potential avenues for future work.

II. THE MODEL

A. SOME EXTENSIONS OF THE BLANCHARD (1985) MODEL

The following analysis presents an extended version of the Blanchard (1985) model, characterizing the dynamic behavior of an economy where agents have finite horizons. The current extensions involve the inclusion of liquidity-constrained consumers to the basic framework and the addition of life-cycle earnings profiles to allow for age-dependent income.⁵

⁴See Bernheim and Bagwell (1986) for a critique of the "dynastic" framework.

⁵Blanchard himself admits, "The main drawback of this approach is that it captures the finite horizon aspect of life but not the change in behavior over life, the 'life-cycle' aspect of life. In that respect it is closer to the initial formulation of permanent income by Friedman (1957) than to that of life cycle by Modigliani (1966) It is poorly adapted to issues where differences in the propensity to consume across agents is potentially important" (Blanchard (1985), p. 224).

These features, in addition to population growth, increase the non-Ricardian properties of the model so that fiscal policy and public debt have a significant impact on real economic activity.

B. THE BASIC MODEL

Following Blanchard (1985),⁶ we consider an economy populated by finitely lived agents, each facing a constant probability p of dying at each moment throughout his or her life. Life expectancy—representing an agent's planning horizon—or the expected time until death, is given by $1/p$.⁷ If the probability of death goes to zero, agents have infinite horizons.

Although each individual faces an uncertain time of death, the law of large numbers implies that generations decline in size deterministically over time with a fraction p dying at each moment. Also at each point in time, a new generation is born of size normalized to p . Consequently, the number of survivors from a generation born at time s remaining at time t is equal to $pe^{-p(t-s)}$, and the size of the total population—aggregating over all existing generations (indexed by s)—is constant and normalized to unity: $\int_{-\infty}^t pe^{-p(t-s)} ds = 1$.⁸

C. LIQUIDITY CONSTRAINTS

Departing from Blanchard (1985), we further assume that each new generation is born into a "risk pool" and initially denied access to life insurance (capital markets). Consequently, these agents cannot gain immediate access to their expected future wealth and income over their lifetimes. Instead, they are constrained by their current resources. Specifically, we define $i(t)$ as the index denoting the oldest generation still credit-rationed at time t . Assuming that a generation graduates out of the pool just as another is born into it, the fixed proportion λ of liquidity-constrained individuals in the economy is given by:

$$\int_{i(t)}^t pe^{-p(t-s)} ds = 1 - e^{-p(t-i(t))} = \lambda; \quad \frac{di(t)}{dt} = 1. \quad (1)$$

⁶See also Yaari (1965) and Weil (1989).

⁷The probability of being alive t periods ahead is given by e^{-pt} . Equivalently, the time until death is a random variable with an exponential distribution, represented by the density function $f(t) = pe^{-p(t-s)}$, and its expected value is given by $\int_0^{\infty} tf(t) dt = 1/p$.

⁸The model can be easily modified so that the birth rate exceeds the death rate, allowing population to grow. See Buiter (1988).

Equation (1) can have the following economic explanation: because agents do not have complete information, "unproven" younger generations are initially denied access to capital markets until a sufficient number of observations have been obtained regarding their behavior and performance.⁹ In a more general setting, this specification can be motivated in the case where agents can be divided into different unobservable types (creditworthy or not). Older agents with an established credit history have the opportunity simply by virtue of their age to signal their type and overcome the asymmetric information problem.¹⁰ Younger generations, on the other hand, are inevitably more uncertain and thus face some form of credit rationing.¹¹

Another explanation for borrowing constraints facing younger generations may involve institutional factors in modern credit markets. In modern market economies, creditors do not have legal access to the future disposable labor income of the borrower in the case of default. Without the possibility of contract enforcement in the form of indentured service (due to *inalienable* labor income), creditors face a greater risk by lending to agents with insufficient collateral in the form of financial—rather than human—wealth.¹² Consequently, younger generations may initially be denied access to credit until a sufficient level of financial assets has been accumulated. Meanwhile, without the ability to borrow against future income and wealth, these agents are left to consume out of current disposable income.

Alternatively, following Campbell and Mankiw (1989), agents here could also be thought of more generally as "rule-of-thumb" consumers during their early years. Under either interpretation—myopia or liquidity constraints—the consequence is that a proportion of agents in the economy consume out of current rather than permanent income.¹³ For these current-income consumers belonging to generations $s > i(t)$, we have $c(s,t) = y(s,t) - \tau(s,t)$,

⁹Back of the envelope calculations using equation (1) suggest that if 20 percent of the population is constrained, the length of time in the pool is approximately 11 years (for $p=0.02$).

¹⁰See Stiglitz and Weiss (1981) for further discussion on credit rationing and asymmetric information.

¹¹Hayashi (1985), Zeldes (1989) and Jappelli (1990) each find empirical evidence suggesting liquidity constraints are more likely for younger families with lower wealth and savings.

¹²See Buiter (1994) for a further theoretical discussion. See Jappelli (1990) for supporting empirical evidence showing that the probability of being liquidity constrained declines as age, wealth and income increase.

¹³See Flavin (1985) for further discussion on the issue of myopia versus liquidity constraints as explanations for excess sensitivity.

where $y(s,t)$, $c(s,t)$ and $\tau(s,t)$ are labor income, consumption, and taxes for generation s at time t .¹⁴

Meanwhile, older generations who have established themselves can contract with insurance companies to receive (or make) payments contingent upon their death. In the absence of intergenerational bequests, these individuals agree to have all of their net wealth—which may be positive or negative—turned over to the insurance firm at the time of their death, in return for flow transfers at rate p while alive. Specifically, surviving agents from generation $s < i(t)$ with net wealth $w(s,t)$ receive $pw(s,t)$ from the insurance company.¹⁵

The dynamic budget constraint facing permanent-income consumers in the economy is thus given by $\dot{w}(s,t) = [r(t) + p]w(s,t) + y(s,t) - \tau(s,t) - c(s,t)$, where dot variables denote time derivatives (i.e. $\dot{w}(s,t) = dw(s,t)/dt$) and r is the real rate of interest. Subject to this constraint, permanent-income consumers maximize expected utility over their lifetimes:

$$E_t \int_t^\infty \ln c(s,z) e^{-\theta(z-t)} dz = \int_t^\infty \ln c(s,z) e^{-(\theta+p)(z-t)} dz; \quad s < i(t), \quad (2)$$

where $E_t[\cdot]$ represents (rational) expectations conditional on the information set at time t , and θ is the rate of time preference. Also in equation (2), utility is assumed to be logarithmic for convenience.¹⁶ Note that the uncertainty of lifetimes, and hence uncertainty about future consumption, raises the effective discount rate.

Solving the dynamic optimization problem facing permanent-income consumers, we derive the consumption function for these individuals: $c(s,t) = (\theta + p)[w(s,t) + h(s,t)]$, where $h(s,t)$ is a measure of *human wealth*—equal to the present value of future labor income over

¹⁴Consumption equals disposable (labor) income provided that the constraint is binding (desired consumption exceeds current resources). Given that income rises rapidly initially in the earnings profile (see section E), this is likely the case. Otherwise the specification serves as an approximation. The more general specification follows from maximizing equation (2) over the period while constrained subject to a non-negativity constraint on wealth: $w(s,t) \geq 0$. See Zeldes (1989) for a further discussion of the general case explicitly incorporating borrowing constraints to allow for more general savings behavior.

¹⁵This is the zero profit condition for the (perfectly competitive) insurance industry. Insurance firms pay $pw(s,t)$ to each living member of generation s and inherit estates worth $w(s,t)$ from the proportion p of that cohort who die at time t .

¹⁶In order to develop a plausible quantitative model, it is necessary to allow for a broader range of elasticities of substitution across consumption in different periods—see Section III.

the agent's expected lifetime.¹⁷ Hence, agents not faced with borrowing constraints consume a constant fraction of their human and non-human wealth (permanent income) each period.¹⁸ To derive aggregate consumption, we simply integrate individual consumption over all existing generations.¹⁹ Assuming that income and taxes are *not* generation-specific (i.e., $y(s,t)=Y(t)$, $\tau(s,t)=T(t)$), aggregate consumption in the economy is characterized by the following expression:

$$\begin{aligned} C(t) &= (\theta + p)[W(t) + (1-\lambda)H(t)] + \lambda[Y(t) - T(t)], \\ &= C^p(t) + C^c(t). \end{aligned} \tag{3}$$

In equation (3), W and H represent total financial and human wealth, and C^p and C^c represent aggregate consumption across permanent and current-income consumers.

Note that permanent-income consumers who make up $(1-\lambda)$ of the population, hold $(1-\lambda)$ of the human wealth but hold *all* of the financial wealth in the economy, because every individual is born without assets and newer generations do not save initially while liquidity-constrained. Thus, consumption for individuals with access to insurance is proportional to their aggregate permanent income: $W + (1-\lambda)H$. If $\lambda=0$, the model collapses back to Blanchard (1985).

In equation (3) note that λ can be interpreted as the degree of *excess sensitivity* of consumption to current disposable income compared to the case where every agent consumes according to the permanent income hypothesis (no current-income consumers). In support of this modification, a substantial number of studies have documented the excess sensitivity hypothesis as an empirically significant departure from the implications of the strict permanent

¹⁷ $h(s,t) \equiv \int_t^\infty [y(s,v) - \tau(s,v)] e^{-\int_t^v [r(u)+p] du} dv$.

¹⁸ Under the general class of CRRA utility, the marginal propensity to consume (mpc) out of wealth depends on the interest rate. There, the consumption function is given by:

$c(s,t) = \Delta^{-1} [w(s,t) + h(s,t)]$, where the inverse of the mpc evolves according to:

$\dot{\Delta} = [(\sigma - 1/\sigma)r(t) + p + \theta/\sigma]\Delta - 1$.

¹⁹ Aggregate variables, denoted by capital letters, are derived by integrating over generations (indexed by s): $X(t) \equiv \int_{-\infty}^t x(s,t) p e^{-p(t-s)} ds$.

income hypothesis as described by Hall (1978).²⁰ In turn, liquidity constraints have been widely acknowledged as the most likely source of that excess sensitivity.^{21 22}

Moreover, equation (3) further shows that for the simple case in which disposable income is *not* generation-specific, the proportion of current-income consumers in the economy also equals the degree of sensitivity of consumption to current income. However, this result does not apply to the case when income is age-dependent, as will be shown when earnings profiles are introduced. In general, it is the proportion of disposable income rather than the proportion of the population associated with liquidity-constrained individuals that determines the degree of sensitivity of consumption to current income. We will return to this point shortly.

First, aggregating over each individual's human wealth, the dynamics for total human wealth can be represented (dropping the time index) as follows:

$$\dot{H} = [r+p]H - [Y - T]. \quad (4)$$

Meanwhile, the evolution of aggregate financial wealth reflecting a dynamic budget constraint for the overall economy is given by:

$$\dot{W} = rW + (Y - T) - C. \quad (5)$$

In the simple, closed-economy case, financial wealth consists of domestic equity and bond holdings. Given that total private (net) wealth is defined by $W=K+B$, equation (5) can be

²⁰For empirical evidence on the excess sensitivity of consumption to disposable income see Flavin (1981), Hall and Mishkin (1982), Jappelli and Pagano (1989), Campbell and Mankiw (1989) and the references presented therein.

²¹Jappelli and Pagano (1989), for example, present evidence of capital market imperfections and credit rationing underlying the excess sensitivity of consumption to current consumption in a cross section of countries. See also Flavin (1985), Campbell and Mankiw (1989). See Hayashi (1987) for a survey of studies on liquidity constraints.

²²On a related note, excess sensitivity of consumption to *anticipated* changes in income is closely related to the issue of excess *smoothness* of consumption to *unanticipated* innovations in income as departures from the predictions of the strict PIH model. See Campbell and Deaton (1989). These issues are in fact aspects of the same phenomenon and can be generated by liquidity constraints (excess sensitivity hypothesis). See Flavin (1993).

expressed as the sum of two separate equations for capital and debt accumulation, respectively.

On the production side, all factors are assumed to be paid their marginal products, and the real interest rate (marginal product of capital) adjusts to equate domestic saving and investment. Correspondingly, as implied by the economy-wide budget constraint, the accumulation of capital (abstracting from depreciation) is given by:

$$\dot{K} = F(K) - G - C, \quad (6)$$

where $F(K)$ is the concave, twice-differentiable aggregate production function (note: labor L is normalized to 1). As for the public sector, debt accumulation and the government's dynamic budget constraint is given by:

$$\dot{B} = rB + G - T, \quad (7)$$

where B is the stock of public debt. In equation (7), the primary deficit plus interest payments on the existing stock determines the government's bond-financing requirements and the corresponding rate of debt issue.

D. SOME STEADY-STATE ANALYSIS

In this section we conduct some comparative steady-state analyses based on the aggregate dynamics described in equations (3)-(7). First, we can show the failure of Ricardian equivalence.²³ Consider the effects of a change in the timing of taxes (i.e., a change in debt) holding the level of government spending constant ($dG=0$): based on the stationary system of equations, the effects of an increase in the steady-state level of debt on the steady-state capital stock can be represented as:

$$\frac{d\bar{K}}{d\bar{B}} = \frac{(1-\lambda)(p(\theta+p)) + \lambda(\bar{r}+p)(\theta+p-\bar{r})}{(1-\lambda)\left[\frac{d^2F}{dK^2}\bar{C} + (\bar{r}-\theta) - p(p+\theta)\right] - \lambda(\bar{r}+p)(\theta+p-\bar{r})} < 0, \quad (8)$$

²³For a discussion of possible factors undermining Ricardian equivalence, see Bernheim and Bagwell (1988).

where bar variables denote long-run values. Equation (8) shows that an increase in the level of debt crowds out the long-run capital stock, provided that net private wealth, disposable income, and hence consumption are non-negative in the steady state²⁴, and given that the long-run equilibrium rate of interest satisfies: $\theta < \bar{r} < \theta + p$.²⁵

The basic intuition behind this result follows from the uncertainty of lifetimes which creates a wedge in private versus public discount rates. Consequently, the choice between tax-financing versus bond-financing influences private consumption decisions, because agents face the possibility of not being around to bear future tax burdens. More to the point, with a positive *birth* rate—at the replacement level equal to the death rate—the government's future tax base remains constant, whereas an individual's expected future resources and tax burdens decline at rate p . Hence, agents view a partial shifting of the incidence of impending taxes to future generations of new taxpayers.²⁶ The end result is that an increase in debt displaces capital in the portfolio of private agents. In other words, changes in public dissavings are not completely offset by changes in private savings.

Adding in liquidity constraints, the short- and long-run effects of this fiscal policy experiment are even larger due to the excess sensitivity of consumption to current disposable income. Changes in debt affect the consumption of permanent-income consumers as before, but also impact directly on consumption of current-income consumers by affecting contemporaneous net-of-tax income in direct proportion to the change in taxes. In effect, the government is initially borrowing on behalf of those individuals who cannot, thereby raising their current consumption temporarily. Ultimately, however, the larger short-run impact of fiscal policy on aggregate consumption (and capital accumulation) leads to a further reduction in the long-run capital stock and lower steady-state consumption than in the absence of liquidity constraints.

²⁴The long-run relationship between wealth, income and consumption is given by:

$$\bar{K} + \bar{B} = (1 - \lambda) \left[\frac{\bar{r} - \theta}{(\bar{r} + p)(\theta + p - \bar{r})} \right] (\bar{Y} - \bar{T}), \text{ and } \bar{C} = \left[\frac{p(p + \theta)(1 - \lambda)}{(\bar{r} + p)(\theta + p - \bar{r})} + \lambda \right] (\bar{Y} - \bar{T}).$$

²⁵The proof that the real interest rate lies between the rate of time preference and the effective discount rate follows exactly as in Blanchard (1985). Note that the condition that $\bar{r} > \theta$ implies that the economy is dynamically efficient in this case.

²⁶Weil (1989) and Buiter (1988) both show that a positive birth rate is the critical factor underlying the absence of debt neutrality (given annuities markets). Buiter in fact shows that a positive death rate (probability of death) is neither necessary nor sufficient. For example, with a zero birth rate and a positive death rate (shrinking population), agents face a growing per capita tax bill in the future which offsets the uncertainty of lifetimes and restores Ricardian equivalence. In conjunction, Weil (1989) shows that with a positive birth rate and a zero death rate (infinite horizons), Ricardian equivalence again fails to hold.

The steady-state welfare implications of fiscal policy also have interesting distributional consequences across generations in the context of liquidity constrained individuals. In the absence of current-income consumers, an increase in fiscal debt crowds out capital formation, thereby raising the long-run real interest rate and lowering the steady-state level of consumption. Hence, the distributional impact of a positive debt shock is to increase the consumption of existing generations (in transition) at the expense of future generations (in steady state) responsible for the higher tax burden and left with a smaller capital stock and less output than otherwise.

With liquidity constraints, a postponement of taxes may also affect welfare across *existing* generations (in steady state).²⁷ In particular, an increase in public debt may actually *increase* steady-state consumption for permanent income-consumers at the expense of current income consumers. Algebraically, this effect is captured by the following expression:

$$\frac{d\bar{C}^p}{d\bar{B}} = \lambda \left[\frac{d\bar{K}}{d\bar{B}} \frac{d^2 F}{dK^2} + \bar{r} \right] + \bar{r} \frac{d\bar{K}}{d\bar{B}} \quad (9)$$

In equation (9), the first term on the right is positive while the second term is negative.²⁸ Hence, the effects of debt on consumption for permanent-income consumers may be either positive or negative, depending (in part) on the size of λ , whereas consumption for current income consumers and aggregate consumption must fall. In other words, even if permanent-income consumers were around to pay the higher taxes levied to stabilize the government's debt position, they might nevertheless enjoy higher consumption (across steady states) provided there were enough liquidity-constrained consumers. The reasoning follows from the fact that since consumers with access to borrowing hold all the financial wealth in the economy (including debt), they fully enjoy the addition to private wealth accompanying an increase in bonds, but bear only part of its eventual tax liabilities which fall on the total population. Hence for those agents, government bonds are clearly net wealth.

Conversely, current income consumers are unambiguously worse off in the future, because their steady-state consumption falls with an increase in debt. Although liquidity-constrained individuals may enjoy temporarily higher consumption following a reduction in

²⁷This analysis is only a comparison between steady states and ignores transitional dynamics which we address later.

²⁸The first term represents a positive (net) wealth effect on consumption from larger bond holdings and a higher interest rate, reflecting a transfer of consumption from income-constrained to wealth-constrained individuals. The second term measures a negative output on consumption effect due to less capital and lower output as a result of higher debt.

taxes, they must eventually bear some of the burden of higher taxation without the benefit of holding some of the debt as an asset.

E. EARNINGS PROFILES

To further incorporate generation-specific characteristics, we can also incorporate typical life-cycle profiles for labor income into the analysis. Specifically, to introduce the fact that income tends to increase when an individual is young before eventually declining with age and retirement, we approximate the hump-shaped profile of earnings over an individual's lifetime through time-varying weights on aggregate income equal to the sum of two exponential functions:²⁹

$$y(s,t) = \left[a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} \right] Y(t); \quad a_2 < 0, a_1 > 1, \alpha_2, \alpha_1 > 0. \quad (10)$$

In the presence of age-dependent income (and taxes)³⁰ of the form seen in equation (10), the system of equations governing the dynamic behavior of the economy can be summarized as follows:

$$C = (\theta + p) [W + \beta(1 - \lambda_1)H_1 + (1 - \beta)(1 - \lambda_2)H_2] + (\beta\lambda_1 + (1 - \beta)\lambda_2)[Y - T] \quad (11)$$

$$H = \beta H_1 + (1 - \beta)H_2 \quad (12)$$

$$\dot{H}_1 = [r + p + \alpha_1]H_1 - [Y - T] \quad (13)$$

²⁹The weights are assumed to be non-negative, requiring that $a_1 \geq -a_2$ and, for $t > s$, $-a_1/a_2 \geq e^{-(\alpha_2 - \alpha_1)(t-s)}$. The exponential expression in the second condition is bounded provided that $\alpha_2 > \alpha_1$, which we assume to be the case. By an adding-up constraint, we also require that $\frac{a_1 p}{\alpha_1 + p} + \frac{a_2 p}{\alpha_2 + p} = 1$.

³⁰In other words, we assume that individual lump-sum taxes $\tau(s,t)$ —relative to aggregate taxes $T(t)$ —follow the same hump-shaped distribution pattern across generations as that for (gross) labor income seen in equation (13).

$$\dot{H}_2 = [r+p+\alpha_2]H_2 - [Y-T] \quad (14)$$

With generation-specific income, the sensitivity of consumption now depends on the relative share of aggregate disposable income held by current-income consumers seen by the coefficient $\beta\lambda_1 + (1-\beta)\lambda_2$ in equation (14).³¹

Depending on the shape of the earnings profile (see Section III.B), the degree of excess sensitivity in consumption may be significantly larger than in the simplest case when income is not age-dependent and the degree of sensitivity just equals the proportion who are liquidity constrained. In particular, one can show that the faster the rate at which income eventually declines with age (larger the α 's), the greater is the proportion of disposable income held by younger generations. Intuitively, as income falls off faster for older generations, the relative share of income earned by younger generation must rise given aggregate income. In the case of monotonic declining income—i.e., $\alpha_1 > 0$, $\alpha_2 = 0$ —examined in Blanchard (1985), the income distribution is skewed even further toward the young and the corresponding degree of excess sensitivity becomes even greater.

Another important implication of life-cycle income for the non-neutrality of public debt is increasing the wedge between public and private discount rates. With eventually declining labor income, private agents have a higher effective discount rate on future disposable income, implicit in equations (13) and (14).³² With lower income in the future, individuals across generations tend to reduce consumption and accumulate more capital to finance consumption later in life when income is low. In effect, there is an additional saving-for-retirement motive.³³ As for the real effects of debt financing, facing a declining income profile over their lifetime, agents further discount the impact of future tax liabilities as the prospective tax base shifts further to future younger generations with higher taxable income.

³¹We define $\beta \equiv \alpha_1 p / (p + \alpha_1)$, $\lambda_1 \equiv 1 - e^{-\alpha_1(t-i(t))}$, $\lambda_2 \equiv 1 - e^{-\alpha_2(t-i(t))}$. Note that $\lambda_2 > \lambda_1$ by definition, and both measures are greater than λ for plausible income profiles. We return to this issue in the calibration section to follow.

³²Integrating up equation (13), for example, (and imposing a transversality condition) yields the definition of H_1 : $H_1(t) \equiv \int_t^\infty [Y(v) - T(v)] e^{-\int_t^v [r(u) + p + \alpha_1] du} dv$.

³³See Blanchard (1985). From the point of view of aggregate dynamics, a higher effective discount rate raises the speed of adjustment or convergence rate to the steady state.

III. THE LONG-RUN EFFECTS OF GOVERNMENT DEBT

While the analytics presented in the previous section suggest that adding liquidity constraints and age-dependent income processes to the Blanchard model will make the model less Ricardian, we have not yet provided any estimates of how much the predictions of the standard model are likely to change when we incorporate more realistic assumptions about lifetime income profiles. The main purpose of this section is to embed a more realistic calibration of the model into a simple closed-economy model and then to examine the effects on the long-term predictions of the model. As we mentioned in the introduction, Evans (1991) has demonstrated that Ricardian equivalence holds approximately in the Blanchard model. For this reason, it is probably worthwhile to start with his calibration of the model and explain why the effects on the real interest rate are so small.

A. EVANS'S CALIBRATION OF THE BLANCHARD MODEL

In his base-case calibration of the Blanchard model, Evans (1991) assumes logarithmic utility, zero population growth, and a steady-state productivity growth rate of 3 percent per annum.³⁴ He then investigates the comparative statics of the model for various assumptions about the length of planning horizons ($1/p$), the elasticity of substitution ($1/\sigma$), and the rate of time preference (θ). Since our model is a generalization of the Blanchard model it is straight forward to replicate Evans' basic results to show that the model obeys approximate Ricardian equivalence.

The experiment considered by Evans (1991) consisted of a change in the steady-state deficit-to-consumption ratio of 3 percentage points. In the steady state, the deficit-to-consumption ratio will be proportional to the debt-to-consumption ratio, where the degree of proportionality is simply equal to the growth rate of the economy. Without inflation and real growth of 3 percent per annum, his experiment translates into a change in the debt-to-consumption ratio of 100 percentage points. From the perspective of the world economy, this shock would require a run-up in government debt that would be five times larger than what

³⁴For the production side of the economy Evans employs a Cobb Douglas production function and sets capital's share of income equal to 0.25. We use the same production function but we use a slightly higher estimate of capital's share. Our estimate of 0.35 is derived by taking an historical postwar average of data for the Group of Seven major Industrial Countries.

has actually been observed over the last two decades in the OECD countries.³⁵ Hence, we consider a somewhat different set of experiments.

As mentioned, net public debt currently stands around 40 percent of GDP. For an initial benchmark, we compute a steady-state solution assuming that this debt ratio were to stabilize at this level forever. We then compare this steady-state solution with two alternatives. In the first experiment, we ask what would be the long-term benefits of eliminating all government debt in the OECD countries, i.e., a reduction in the debt-GDP ratio of 40 percentage points. In the second experiment, we assume that the debt ratio continues to drift up by an additional 20 percentage points. This second experiment would be very much like repeating the experience over the last two decades.

Following Evans (1991) we assume that the elasticity of substitution ($1/\sigma$) is equal to 1 and that labor productivity grows at constant rate of 3 percent per annum. Given an initial debt ratio of 40 percent, a planning horizon of 50 years ($1/p = 50$, where $p = .02$) and a rate of time preference of 0.011, the steady-state real interest rate is 4.4 percent.³⁶ These numbers are very close to one of the scenarios reported in Table I of Evans (1991). Table I provides our comparative statics for the two experiments described above, as well as for several intermediate cases where we vary the debt ratio in increments of 10 percentage points. According to this calibration of the model, eliminating all government debt in the OECD countries would reduce the real interest rate by 4 basis points and increase output and consumption by 0.13 percent and 0.01 percent, respectively. Despite the fact that agents have finite horizons, the model's predictions are very close to a pure Ricardian model where agents effectively have infinite horizons.

One might expect that these results are sensitive to certain assumptions that have been used to calibrate the model. There is considerable empirical evidence that suggests that consumption is much less responsive to changes in the real interest rate than what is implied by the assumption of logarithmic utility. For example, in a survey paper Blundell (1989) argues that the elasticity of consumption is likely to be less than 0.5. Although there is obvious uncertainty around any estimate of this elasticity, our own reading of the literature suggests

³⁵On page 630, footnote 12, Evans (1991) states that 'because my calculations describe how a closed economy would respond to budget deficits, they more nearly describe how the world economy would respond to budget deficits than how the U.S. economy would.' Presumably, the reason why Evans chose such a large shock is that it was necessary, given the Ricardian nature of the model, to get any significant changes in the real interest rate.

³⁶This baseline solution for the real interest rate on government debt seems plausible. Using data on one-year debt instruments for nine countries with liberalized capital markets, Gagnon and Unferth (1994) report an average real interest rate of 4.2 percent for the sample period covering 1978 to 1993.

that a more reasonable range would be centered around 0.3 instead of one.³⁷ In any event, in order to understand the importance of this assumption it is useful to consider a steady-state consumption function where we allow for a broader range of elasticities of substitution across consumption in different periods. The case of logarithmic utility can be easily generalized to allow for a utility function of the constant relative risk aversion (CRRA) class:

$$U(c(s,t)) = \frac{c(s,t)^{1-\sigma}}{1-\sigma} \quad (15)$$

where σ^{-1} is the (intertemporal) elasticity of substitution ($\sigma=1$ for log utility). In this case, the consumption function in the steady state takes the following form:

$$C = [r(1-1/\sigma) + \theta/\sigma + p] [K + D + (Y - T)/(r + p - g)] \quad (16)$$

This equation simply states that consumption in the steady state will be equal to the marginal propensity to consume times wealth. In the case of our closed economy model, wealth consists of capital (K), government debt (D), and the discounted sum of steady-state after tax labor income ($Y - T$), where g is the growth rate of labor productivity.³⁸ Most of the results in Evans (1991) can be understood by studying the properties of this equation. In the case where Evans considers logarithmic utility ($\sigma=1$), the consumption function becomes:

$$C = (\theta + p)[K + D + (Y - T)/(r + p - g)] \quad (17)$$

Intuitively, the degree of crowding out should depend on the sensitivity of consumption to changes in the real interest rate. If a part of government debt is viewed as net wealth, because consumers excessively discount future tax liabilities, there will be a tendency to 'overconsume' and this will require an increase in the real interest rate to re-establish a new steady-state equilibrium. On the other hand, if consumption is very sensitive to changes in the real interest rate, only a small change in the real interest rate will be required. The small effects

³⁷Hall (1988) argues that it may be even lower than 0.2. Econometric work based on the U.S. and the U.K. time series data by Patterson and Pesaran (1992) suggest that it is somewhere between 0.1 and 0.3. Similar results are found by Attanasio and Weber (1993) using U.K. aggregate time series data although they do find a slightly larger range of when they rely upon U.K. panel data.

³⁸This assumes that all consumers are paid the same real wage. This formulation will change when we allow for more realistic lifetime income profiles.

on the real interest rate and consumption reported in Table 1 and Evans (1991) are not difficult to understand in light of this assumption. Recall, in the initial steady-state baseline the total discount factor $(r+p-g)$ in equation 17 is only 0.034. This has two important implications in the model. First, it implies that the value of human wealth represents most of total wealth since human wealth is about 30 times the value of after-tax labor income. Second, it effectively rules out the possibility that interest rates will ever change by very much because even small changes will result in enormous revaluation effects on human wealth.

In the case where the elasticity of substitution is less than one, there are two offsetting effects on consumption. An increase in the real interest still has enormous effects on human wealth but this effect is offset by an increase in the marginal propensity to consume (see equation 16). Indeed, in the limit as the elasticity of substitution goes to zero, these effects exactly offset each other and the model requires a much larger change in the real interest rate to establish a new equilibrium. Unfortunately, as Evans points out there is an absolute limit in the Blanchard model on the allowable range for the elasticity of substitution, beyond which a negative rate of time preference is required to generate a reasonable baseline real interest rate.

In order to estimate the maximum effects that can be obtained from the growth-augmented Blanchard model along these lines, we reduce the elasticity of substitution to 0.75. This is very close to its lowest permissible value in order to maintain a non-negative rate of time preference. We then recompute the effects of changes in the government debt ratio. These results are reported in Table 2. Not surprisingly, the effects on the real interest rate are slightly larger than in Table 1 but because consumption is still very interest-elastic, only small changes in the real interest rate are required to establish a new steady state. Evans shows that adding liquidity-constrained individuals to this model does not change this basic property as long as liquidity-constrained individuals only make up about 25 percent of the total. This result is not surprising, since the consumption of the individuals who are not constrained is still very sensitive to changes in the real interest rate. As we will show below, the tight restrictions on the parameter space in the Blanchard model can be relaxed completely when we employ more 'realistic' assumptions about life-cycle income profiles.

B. THE BLANCHARD MODEL WITH AGE-DEPENDENT WAGE INCOME

In most quantitative applications of the Blanchard model, it is usually assumed that all agents make the same real wage and this real wage grows at the rate of productivity growth (g). With significant productivity growth, this would imply a very unrealistic lifetime income profile. For example, Figure 2 produces the lifetime income profile with Evans' assumption of 3.0 percent productivity growth. Assuming that individuals expect to live 50 years, this implies that individuals who enter the labor force at age 20 can expect to earn 338 percent more when they reach age 70. Moreover, given that the probability of death in the Blanchard model is independent of age, this implies that individuals at age 50 also expect to earn 338 percent more when they reach the age of a 100, and so on. Obviously, if these assumptions had innocuous implications for the results it would be a small price to pay for

analytical tractability. However, as we will demonstrate below, this is not the case. Figure 2 also shows the path of a non-linear lifetime earnings profile (with 1.5 percent productivity growth) when we employ the two exponential functions described in Section I.5 (for $\alpha_1 = .06$, $\alpha_2 = .10$).³⁹ These parameters were chosen to approximately replicate the empirical lifetime income profiles reported by Japelli and Pagano (1990).⁴⁰ This specification of lifetime income allows a more realistic estimate for the elasticity of substitution without imposing negative rates of time preference on the model. Indeed, to generate the same initial control solution of 4.4 percent for the real interest rate, the model now requires a rate of time preference of .045. Table 3 reports the results for the same government debt shocks that were considered earlier. In this case, a reduction in the steady-state government debt from 40 percent of GDP to zero reduces the real interest rate by 122 basis points. In this case the long-run crowding-out effects of debt are quite large, reducing the capital stock by 14.4 percent and the level of output by 4.8 percent. These effects work to shrink the consumption possibilities frontier of the economy; aggregate consumption in the steady state is 2.6 percent higher. These results differ sharply from the results reported in Table I and II. The model no longer is consistent with the property of approximate Ricardian equivalence.

C. THE BLANCHARD MODEL WITH LIQUIDITY CONSTRAINTS

Since the seminal work of Hall (1978), there has been an extensive empirical literature that has provided formal econometric tests of the permanent income hypothesis. Almost of all this work has confirmed Hall's contention that the pure form of the permanent-income hypothesis is not supported by the data. Most of this empirical literature has typically relied upon liquidity constraints to explain the excess sensitivity of consumption to changes in

³⁹Here, we depart from Evans' assumption of 3.0 percent growth in aggregate labor productivity. The lower estimate of 1.5 percent is more consistent with what most economists currently consider a sustainable or steady-state rate of productivity growth. Over the last two decades, real GDP per worker in the United States has increased, on average, by less than 1 percent per annum.

⁴⁰The simplicity of this functional form makes it difficult to replicate all lifetime income profiles that one might want to consider, but is meant to approximate the fact that labor income eventually declines with age. This is a critical assumption for estimating the crowding-out effects in models that assume a constant probability of death. In fact, it is the hazard of living in this model that distinguishes it from a true-blue overlapping generations model. In an earlier version of this paper, one individual suggested that the peak of our lifetime income profile may come too early. It is possible to shift the peak to the right but would imply 'unrealistic' tax transfers to the elderly. Figure 2 is meant to illustrate that the effects of such modifications are small relative to the gross approximation error that is made when it is assumed that everyone's wage grows at a constant rate over their entire lifetime. Section D discusses the sensitivity results.

disposable income. Estimates of excess sensitivity based upon time series data typically vary between 0.2 and 0.6—see, for example, Campbell and Mankiw (1989), Japelli and Pagano (1989), and Patterson and Pesaran (1992). Although econometric tests that have relied upon aggregate time series data have been able to uncover significant indications of liquidity constraints, they do not tell us what types of individuals do not have access to credit markets. In a study that relied upon U.S. household data from the Survey of Consumer Finances, Japelli (1990) finds that liquidity constraints are much more binding for young individuals than they are for older individuals.⁴¹ Based on this detailed evidence and the aggregate time-series evidence on excess sensitivity, we assume that 20 percent of the population is liquidity constrained.⁴² Given our assumptions about lifetime income profiles, this assumption implies that individuals are liquidity constrained for about the first 11 years of their working life.⁴³ In this case, the degree of excess sensitivity in consumption is significantly larger than in the simplest case when income is not age-dependent. Indeed, with this income profile and our assumption that 20 percent of the working age population is constrained for the first 11 years of their working lives, the model generates reduced-form estimates of excess sensitivity equal to around 0.37. Thus, our combined assumptions about liquidity constraints and lifetime income profiles are consistent with the reduced-form estimates of excess sensitivity that are reported in the literature.

Again we reconstruct the baseline solution by computing the rate of time preference that is consistent with a real interest rate of 4.4 percent.⁴⁴ Table 4 reports the effects for the debt shocks considered earlier. Unlike Evans (1991), adding liquidity-constrained consumers

⁴¹Furthermore, this result is confirmed by one of his other findings that shows that accessibility to credit markets is positively related to the level of financial wealth. This result seems quite plausible since, by definition, older individuals have had more time to accumulate financial wealth.

⁴²It is important to note that credit restrictions in the U.S. are probably less severe than in other countries. The model under consideration is meant to represent the world economy. For evidence on the severity of liquidity constraints across countries see Haque and Montiel (1987).

⁴³This assumption is not inconsistent with the fact that young individuals borrow to purchase consumer durables. Indeed, the restrictions that banks impose on such loans is a good indication of the severity of credit restrictions since such loans are invariably tied to the individual's current disposable income. Individuals with very low current disposable income but good future income prospects are typically rejected before any formal adjudication process begins. And even when loans are granted to young individuals, the required payback period is, on average, much shorter than the life of the asset.

⁴⁴This baseline solution assumes lifetime income profiles, aggregate productivity growth of 1.5 percent, a rate of time preference of 0.075 and an elasticity of substitution equal to 0.30.

into the model has significant effects on the comparative statics of the model. In this case, real interest rates are predicted to fall by 147 basis points if government debt was completely eliminated in all of the OECD countries. Still, the marginal effects on the model are small when compared to the effects of adding more realistic lifetime income profiles.

D. SENSITIVITY ANALYSIS

The results in the previous section show that a more realistic calibration of Blanchard's model suggests that there may be significant long-term crowding effects of government debt. There are other avenues that may further increase the effects of government debt. For this paper, we have followed Evans (1991) and assumed that steady-state growth is driven only by gains in productivity. Weil (1989) and Buiter (1988) have shown that the fundamental assumption that breaks down strict Ricardian equivalence in the Blanchard model is a positive birth rate; in particular, a higher birth rate makes the model less Ricardian.⁴⁵ Although a stationary population for the world may be appropriate for an analysis of the very long run, it is still interesting to ask how the comparative statics of the model would change in a world with positive population growth. In order to do this, we repeated the experiment reported in Table 4, except this time we assumed that steady-state population growth was 1 percent per annum. In this case, a decline in the debt-GDP ratio of 40 percentage points reduces the interest rate by 163 basis points, versus the 147 basis points reported in Table 4.

In the base-case calibration of the model, we assumed that only 20 percent of the population was liquidity constrained. The empirical evidence suggests that there may also be a significant group of consumers that simply consume out of disposable income even when they have access capital markets. To get some feel for how this would affect the results, we assumed that an additional 20 percent consumed all of their disposable income. Since this reduces the aggregate interest rate sensitivity of consumption even further, it has some very significant effects on the comparative statics. With this change, a decline in the debt-GDP ratio of 40 percentage points reduces the real interest rate by 200 basis points.

Finally, the choice of the functional form that was employed to approximate life-cycle income was based partly on the desire to keep the model tractable. For this reason, we should be confident that the main conclusions about the importance of crowding out are not overly sensitive to this approximation. Our experiments have confirmed that it is possible to delay the peak in the lifetime income profile for an additional decade without changing the general conclusion that there are significant long-term crowding-out effects from government debt.

⁴⁵The intuition for this result is simple. In a world where current generations are disconnected from future generations, a higher rate of population growth will mean that current generations will expect to pass on a larger share of the tax burden to future generations.

IV. EMPIRICAL ESTIMATES OF THE EFFECTS OF GOVERNMENT DEBT

The calculations that were made in the previous section are useful for providing a general equilibrium perspective of reduced-form time-series evidence that attempts to directly relate government debt and measures of the real interest rate. Although an extensive summary of this evidence is well beyond the scope of this paper, it is worthwhile to provide a few comments. First, evidence based on long historical time series has suggested that the effects of debt on real interest rates may even be perverse. For example, Evans (1985) has concluded that "concern should not focus on what deficits do to interest rates, capital accumulation, or economic growth, for there is precious little evidence that deficits affect these variables."⁴⁶ At the other end of the spectrum, Ford and Laxton (1994) report that the buildup in government debt since the late 1970's was the main culprit behind the increase in real interest rates in virtually all countries. Indeed, their empirical work suggests that the increase in the aggregate measure of OECD net public debt since 1978, can account for an increase in real interest rates of between 250 and 400 basis points.

V. CONCLUSIONS

In this paper, we have shown that including more realistic individual lifetime income profiles and liquidity constraints can significantly change our estimates of the long-term effects of government debt. OECD estimates of aggregate net public debt for 18 of the largest industrial countries show an increase of about 20 percentage points in the debt-GDP ratio since the late 1970s. Based on our estimates, this increase can be expected to result in a permanent increase in the world real interest rate of 76 basis points. In terms of world output our estimates imply that this increase in government debt will result in a permanent output loss of 2.9 percent per annum. Although these estimates are smaller than what is suggested by recent reduced-form regression estimates, they are clearly large enough that policymakers should be concerned about the long-term implications of government debt. We have assumed throughout this paper that a higher level of government debt could be financed with nondistortionary lump-sum taxation. However, in reality, governments rarely have recourse to lump-sum taxation. To the extent that distortionary taxes reduce the capital stock and raise the marginal product of capital, there may be further significant effects on real interest rates and output.

There are several interesting extensions of this paper. First, the closed-economy framework could be easily extended to a two-country or three-country world to examine the externalities imposed on other countries from increasing government debt. Second, the model could be extended to allow for distortionary taxation. This would hopefully provide a better perspective about why real interest rates have risen in virtually all countries.

⁴⁶See Evans (1985) page 86.

**Table 1. Steady-State Effects of Government Debt in Evan's (1991)
Calibration of the Blanchard Model**

Change from an Initial Steady State with a Debt-to-GDP Ratio of 40 Percent

	Debt-to-GDP Ratio						
	0	10	20	30	40	50	60
Output (in percent)	0.13	0.10	0.06	0.03	0.00	-0.03	-0.06
Consumption (in percent)	0.01	0.01	0.01	0.00	0.00	-0.00	-0.01
Capital stock (in percent)	0.51	0.38	0.25	0.13	0.00	-0.13	-0.25
Real interest rate (Basis points)	-4	-3	-2	-1	0	1	2

**Table 2. Steady-state Effects of Government Debt in Evan's (1991)
Calibration of the Blanchard Model
(With the elasticity of substitution set at its minimum of .75)**

Change from an Initial Steady State with a Debt-to-GDP Ratio of 40 Percent

	Debt-to-GDP Ratio						
	0	10	20	30	40	50	60
Output (in percent)	0.20	0.15	0.10	0.05	0.00	-0.05	-0.10
Consumption (in percent)	0.09	0.06	0.04	0.02	0.00	-0.02	-0.04
Capital stock (in percent)	0.57	0.43	0.29	0.14	0.00	-0.29	-0.56
Real interest rate (Basis points)	-5	-4	-3	-1	0	1	3

**Table 3. Steady-State Effects of Government Debt in the
the Model with more Realistic Lifetime Income Profiles**

Change from an Initial Steady State with a Debt-to-GDP Ratio of 40 Percent

	Debt-to-GDP Ratio						
	0	10	20	30	40	50	60
Output (in percent)	4.83	3.57	2.35	1.16	0.00	-1.13	-2.25
Consumption (in percent)	2.59	1.94	1.30	0.65	0.00	-0.65	-1.30
Capital stock (in percent)	14.43	10.55	6.87	3.35	0.00	-3.21	-6.28
Real interest rate (Basis points)	-122	-92	-62	-31	0	31	63

**Table 4. Steady-State Effects of Government Debt in the Model with more Realistic
Lifetime Income Profiles and Liquidity-Constrained Individuals**

Change from an Initial Steady State with a Debt-to-GDP Ratio of 40 Percent

	Debt-to-GDP Ratio						
	0	10	20	30	40	50	60
Output (in percent)	5.90	4.40	2.92	1.45	0.00	-1.43	-2.85
Consumption (in percent)	3.12	2.37	1.60	0.81	0.00	-0.82	-1.66
Capital stock (in percent)	17.78	13.09	8.56	4.20	0.00	-4.04	-7.93
Real interest rate (Basis points)	-147	-112	-76	-39	0	39	81

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