Estimating GPM with Dynare

I. Ermolaev, C. Freedman, M. Juillard, O. Kamenik,
D. Korshunov, D. Laxton

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Overview

- Bayesian estimation basics
- What Dynare calculates
- Structure of Dynare 4 input file(s)
- Dynare prior
- Dynare posterior
- Common errors
- GPM model file

Bayesian Estimation Basics

- Bayesian approach to inference
- Bayes formula
- Model
- Prior
- Bayes formula decoded

Bayesian Approach to Inference

Bayesian inference helps to sort out our goals

- Wrong goals:
  - Give me true parameter values of my true model
  - Give me true parameter values and use NO model
  - Give me confidence intervals assuming repeated experiments
- Good goals:
  - What information about parameters in my model is conveyed by observed data
  - Want to use a model even if I know some parameters are not identified
  - Want to use a model even if I know it is wrong in some aspects
  - Want to put together prior information and information in the data

Bayes Formula

Nomenclature:

- Model M
- Data Y
- Parameters θ

Bayesian inference uses three source of information:

1. Model: likelihood $L(Y|\theta, M)$ and prior $p(\theta|M)$
2. Observed data Y
3. Loss function associated with our decision $\phi(\theta, \delta)$

The central Bayes formula

$$p(\theta|Y, M) = \frac{p(Y|\theta, M)p(\theta|M)}{p(Y|M)}$$

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What is A Model?

- Model is an implicit probabilistic relation among various random variables
- If we divide the variables to parameters and data, then we can say that given the parameters, a model defines density of the data $L(Y|\theta, M)$
- Note that the division between data and parameters is arbitrary
- Both data and parameters are random variables
- Model, although often a mathematical object, is subjective
- In other words, a model is a human choice

What Is A Model? Likelihood continued

Likelihood is purely defined by the model. Examples:

- The Gaussian model $y \sim N(\mu, \sigma^2)$ gives the likelihood:

$$L(y|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(y-\mu)^2}$$

- VAR model $y_t = A_1 y_{t-1} + A_2 y_{t-2} + \epsilon_t$, where $\epsilon_t \sim N(0, \Sigma)$ yields

$$L(y_1 \dots y_T|A_1, A_2, \Sigma) = \prod_t \frac{1}{\sqrt{|\Sigma|(2\pi)^p}} e^{-\frac{1}{2}\tilde{\epsilon_t}^T \Sigma^{-1} \tilde{\epsilon_t}},$$

where $\tilde{\epsilon_t} = y_t - A_1 y_{t-1} + A_2 y_{t-2}$ and p is size of y.

I. Ermolaev, C. Freedman, M. Juillard, O. Kamenik, D. Korshunov, D. Laxton

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What is A Prior?

- It is any information on parameters independent on the data, but depends on the model: $p(\theta|M)$
- Where does it come from?
  - Prior research — parameter distributions from estimations on different datasets
  - Expert judgement — reflects also common sense, etc.
  - Mathematical constraints — ruling out exploding solutions, ruling out square root of negative values
  - Imposing semantics — a trend of a variable is less volatile than the variable
  - Imposing units — in regression of new built roads inches do not matter but miles do
  - Technical priors — prevent a failure of estimation algorithm; the failure might be caused by a lack of information, numerical instabilities, slow convergence in low probability areas

Bayes Formula Decoded

Interpretation of the Bayes formula:

- We are given the likelihood L, which says how data might look like if parameters happen to be θ
- We need the posterior $p(\theta|Y, M)$, this is how parameters might look like if the data happen to be Y
- This is the Bayesian Duality principle, all what is needed is to perform this **inversion** process
- There is no theoretical distinction between parameters and data!
- Additional information comes in form $p(\theta|M)$ – a prior. Information on the parameters independent on the data.

Bayes Formula: Marginal Data Density

What about the denominator $p(Y|M)$? It is called *marginal data density*. It can be calculated as

$$p(Y|M) = \int p(Y|\theta, M)p(\theta|M)\mathrm{d}\theta$$

Where do we need it?

- It does not matter for the inversion, it is only a scaling constant.
- It is important for model evaluation. It gives an average model fit to the data independent on our final choice, i.e. how good is the model structure in explaining the data regardless the parameters.
- Still it depends on the prior!

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Estimation Algorithm in Dynare

Two phases (second optional):

1. The posterior mode θ^{mode} calculations
   - Optimization routine finds minimum θ^{mode} of $-log(p(y|\theta) \cdot p(\theta))$
   - Check plots are drawn to visually check the mode
   - Hessian at the mode gives Laplace approximation of the posterior density
2. The parameters are drawn from the posterior
   - Markov Chain Monte Carlo Metropolis–Hastings sampler is used
   - Multiple chains are simulated
   - A portion of each chain is dropped
   - Chain convergence is checked

Likelihood Evaluation

The model is in the linear (or linearized) form:

$$A(\theta)E_t y_{t+1} = B(\theta)y_t + C(\theta)y_{t-1} + D(\theta)u_t$$

where shocks $u_t \sim N(0, \Sigma(\theta))$. The subvector of observables is denoted y_t^{obs}.
The data likelihood $p(y|\theta)$ is evaluated in three steps:

1. Reduced form $y_t = F(\theta)y_{t-1} + G(\theta)u_t$ is calculated
2. State space is obtained by adding equation $y_t^{obs} = Sy_t + \epsilon_t$, where S is a selection matrix, and ϵ_t are measurement errors (might depend on θ)
3. The likelihood is evaluated with Kalman filter
4. For diffuse initialization of the first period, Dynare uses Exact Diffuse Filter by Durbin and Koopman

Drawing From Posterior Distribution

Dynare uses Markov Chain Monte Carlo Metropolis–Hastings:
Let θ^k be the last draw, let $j(\tilde{\theta})$ be a jump distribution. The next draw in the chain is obtained as follows:

1. Draw $\tilde{\theta}$ from j
2. Calculate $\nu = \min\left\{\frac{p(\theta^k+\tilde{\theta})}{p(\theta^k)}, 1\right\}$
3. With probability of ν, put $\theta^{k+1} = \theta^k + \tilde{\theta}$, otherwise put $\theta^{k+1} = \theta^k$

A selection of the jump distribution influences a rate of convergence. In Dynare, the user scaled Laplace approximation is used as the jump distribution.

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Dynare 4 Input File(s)

- Model file
  - Preprocessor logic
  - Comments
  - Preamble
  - Parameter settings
  - Model section
  - Shock variances
  - Priors
  - Unit roots
  - Observation variables
  - Commands
- Data file
- Steady state file

Preprocessor Logic

What happes if one types `dynare blablah.mod`?

- `dynare.m` file runs preprocessor
- The preprocessor creates `blablah.m`
- `dynare.m` then runs `blablah.m`

Preprocessor does the following:

- Reads `blablah.mod` file and parses variable declarations
- Parses the model equations and creates M-files for derivatives, equation residuals
- Parses other Dynare specific commands and convert them to Matlab
- Everything is put to `blablah.m`
- Other Matlab commands are copied verbatim

If one is not sure what happens, let him check the `blablah.m` file

Comments in Dynare Input File

Dynare preprocessor has two comment constructs:

- Slash-slash construct

```
// a comment to the end of the line
```

- Slash-asterix-asterix-slash construct

```
/* this can be also a
multiline comment */ and here is a
serious code
```

Preamble

Preamble declares endogenous variables, shocks, and parameters

```
var RR_US RR_US_BAR
    UNR_US UNR_US_GAP UNR_US_BAR
    E E2
;

varexo RES_RR_US_BAR RES_UNR_US_GAP
       RES_BLT_US RES_BLT_US_BAR
                  ;

parameters rho_us rr_us_bar_ss
           pietar_us_ss ;
```

Parameter Settings

Parameter settings follow parameter declaration and end with the model section

```
alpha_us1        = 0.7796;
alpha_us2        = 0.1874;
alpha_us3        = 0.3352;
rho_us           = 0.9;
rr_us_bar_ss     = 1.8456;
tau_us           = 0.10;
a                = 1;
b                = 2;
three_no_wonder = a+b;
```

The parameter values given here are used for all calculations before the parameters are changed by the `estimation` command.

Parameter Settings

Model section starts with `model;` keyword and ends with the keyword `end;`

```
model;

UNR_US_GAP = alpha_us1*UNR_US_GAP(-1)
        + alpha_us2*Y_US + RES_UNR_US_GAP;

UNR_US_GAP = UNR_US_BAR - UNR_US;

end;
```

Each equation is delimited by the semicolon, leads are written as `(+8)`, lags as `(-3)`

Shock Variances / Std. Errors

This section defines variance-covariance matrix of the model shocks

```
shocks;

var RES_G_US ;stderr 0.10; // set stderr
var RES_Y_US ;stderr 0.25;
var RES_Y_US,RES_G_US=(.1*0.25*0.1);
   // set covariance
end;
```

The variance-covariance matrix is used for all calculations before it is changed by `estimation` command.

Estimated Parameters

This section declares estimated parameters and assigns them prior distributions

```
estimated_params;
alpha_us1,    beta_pdf,       0.8, 0.1;
alpha_us2,    gamma_pdf,      0.3, 0.20;
stderr RES_BLT_US, inv_gamma_pdf, 0.4 ,inf;
corr RES_Y_US, RES_G_US, beta_pdf,0.25,0.1;
end;
```

Each line defines: parameter, prior shape, prior mean and prior stderr.

Unit Roots

Currently it is necessary to declare unit root (all non-stationary) variables. It is done by a command:

```
unit_root_vars UNR_US_BAR UNR_US LCPI_US
   LGDP_US LGDP_US_BAR BLT_US BLT_US_BAR;
```

We are working on changing Dynare filtering code to relax this requirement.

Observed Variables

We need to declare the list of observed variables and also the trends (if any) of the observed variables.

```
varobs UNR_US RS_US LCPI_US LGDP_US BLT_US;

observation_trends;
LGDP_US (growth_us_ss/4);
LCPI_US (pietar_us_ss/4);
end;
```

The trends must be in `()` paranthesis, and determine the period to period increments.

Observed Variables

After all of that, you can put commands (estimation, simulations)

```
stoch_simul(order=1) Y_US PIE_US4 RS_US;

estimation(datafile=data,nobs=56,
    mode_compute=0,mode_file=soe1_mode,
    mh_replic=0,mh_jscale=0.35);

stoch_simul(order=1) Y_US PIE_US4 RS_US;
```

The first `stoch_simul` command simulates the model using calibrated parameters in `parameters` and `shocks` sections. The second `stoch_simul` command uses the results from `estimation` command.

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Steady State File

Steady state file calculates steady state for Dynare. When needed:

- Model has non-stationary variables
- Model is non-linear and we know the closed form solution for the steady state

The steady state file is a Matlab function:

```
function [ys,check] = soe1_steadystate(ys, junk)
```

Returns ys as a vector of steady state values in Dynare ordering. Trick how to get the ordering:

- Run Dynare without the steady state file
- Print out the contents of `M_.endo_names`

Trick how to get a parameter value:

```
gr_us_ss = get_param_by_name('gr_us_ss');
```

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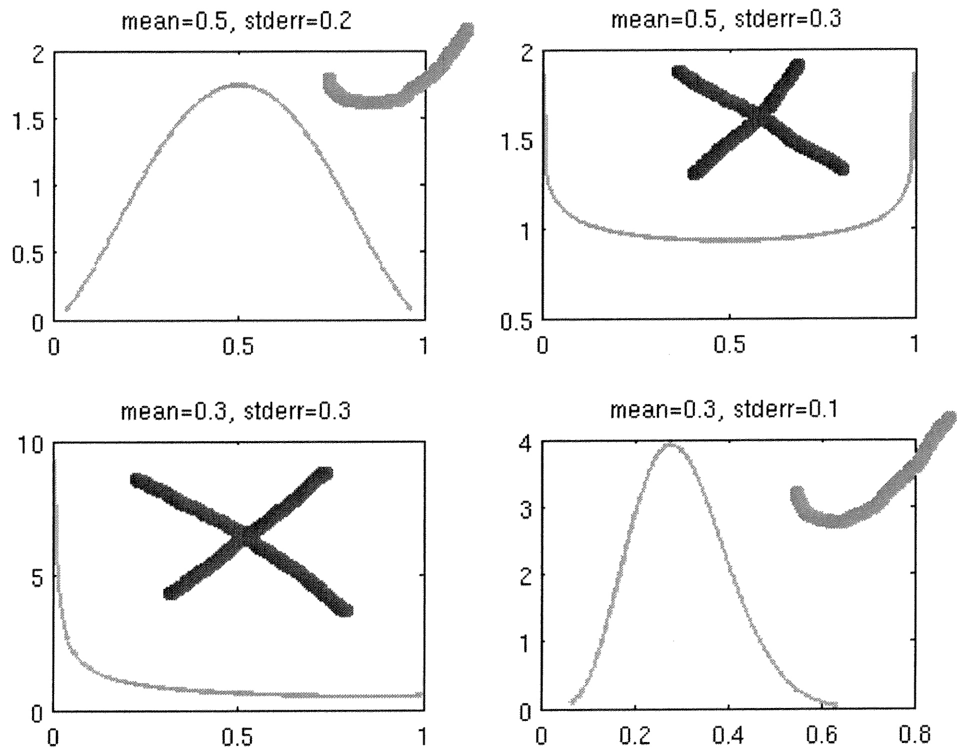
Possible prior shapes

- Normal: keyword `normal_pdf`
- Beta: keyword `beta_pdf`
- Gamma: keyword `gamma_pdf`
- Inverse Gamma: keyword `inv_gamma_pdf`
- Uniform: keyword `uniform_pdf` (I don't like this one)

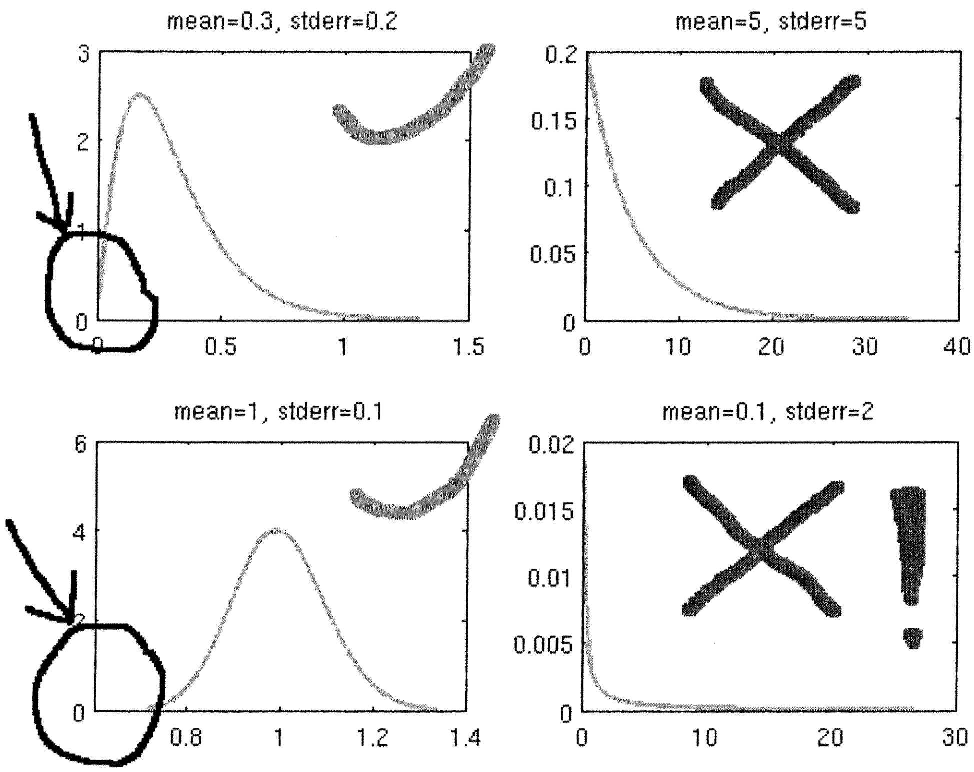
Beta Shape Prior

Used for parameters between zero and one.



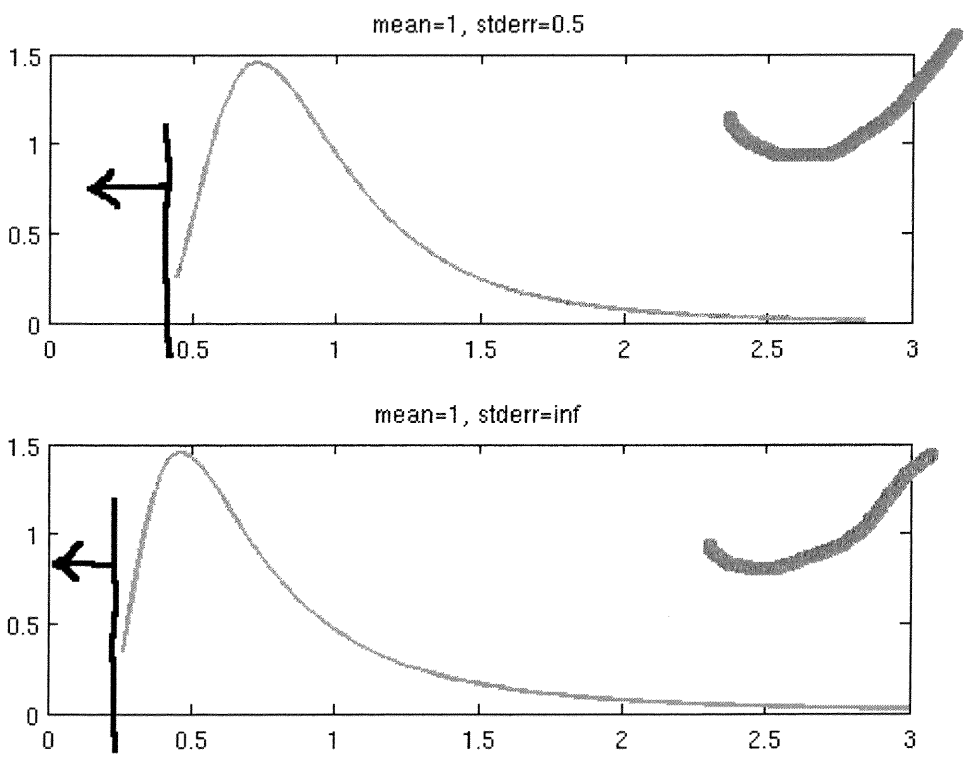
Gamma Shape Prior

Used for parameters which should be positive



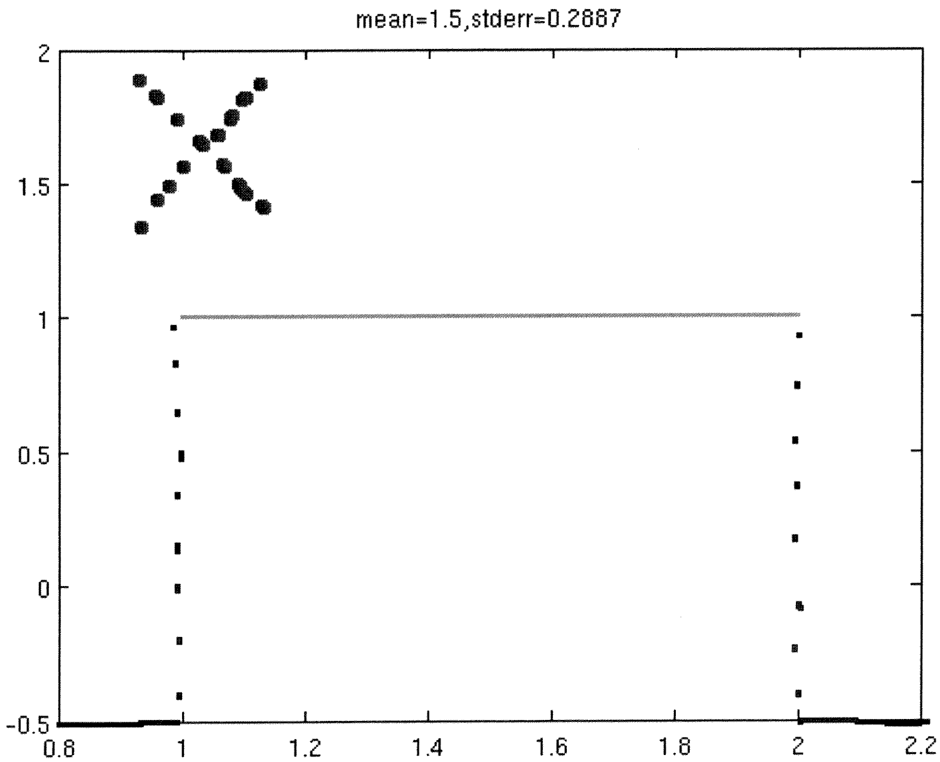
Inverse Gamma Shape Prior

Used for standard errors of shocks allowing for sensible distributions of relative shock sizes



Uniform Shape Prior

The uniform prior does not help to prevent boundary hitting problems



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Dynare Posterior Mode

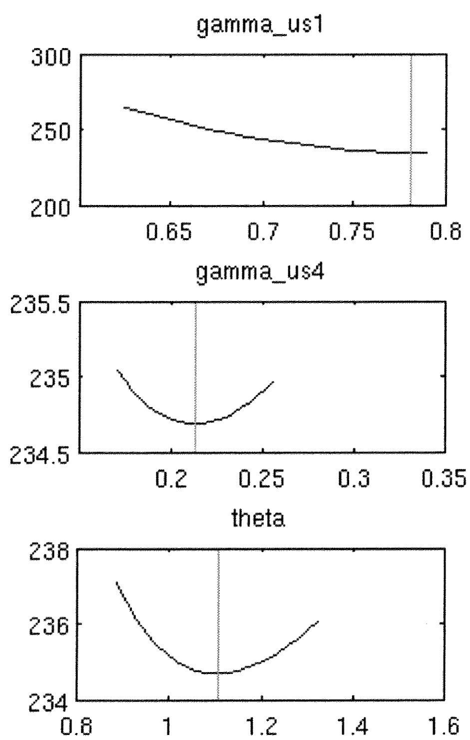
Output from the posterior maximization

```
RESULTS FROM POSTERIOR MAXIMIZATION
parameters
           prior mean     mode    s.d. t-stat prior pstdev

   alpha_us1   0.800   0.8800  0.0472 18.6361 beta  0.1000
   alpha_us2   0.300   0.1700  0.0285  5.9574 gamm  0.2000
standard deviation of shocks
           prior mean     mode    s.d. t-stat prior pstdev

RES_UNR_US_GAP   0.200   0.0996  0.0164  6.0828 invg   Inf
RES_UNR_US_BAR   0.100   0.0473  0.0201  2.3564 invg   Inf

Log data density [Laplace approximation] is -293.142813.
```

And also the variable `oo_`.

Dynare Posterior Mode: Check Plots

Vertical line: mode, smiles: minus log posterior density



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We will go through the Dynare model file for GPM.

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