Divide and Conquer Strategies for Solving Nonlinear Models

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March 5, 2008

This Session

1. General Strategy for Finding the Steady State of a Nonlinear Model

2. Understanding Newton Methods

3. Simple Examples in TROLL

4. Finding a Steady State in Large DSGE Models

Philosophy

Inch by Inch its a Cinch

But Yard by Yard it May be too Hard

General Strategy for Finding the Steady State of a Nonlinear Model

We want to find solutions to a function

$$\mathsf{F}(\mathsf{Y},\mathsf{Z},\theta) = 0$$

where Y are endogenous variables, Z are exogenous variables and θ are a set of parameters. But, the function may include some nasty terms so we need a basic strategy for finding solutions that works.

General Strategy for Finding the Steady State of a Nonlinear Model

What are some options for finding a solution to

$$\text{F}(\text{Y},\text{Z},\boldsymbol{\theta}) = \mathbf{0}$$

1. Choose an algorithm, a function, your desired exogenous variables and parameters. Then let the computer go to work and if it doesn't work come up with better starting values until it does work. This can be a very time consuming process. Some people have described it as the work of a madman!

2. Use a Simple Divide and Conquer Strategy!

Divide and Conquer Strategy

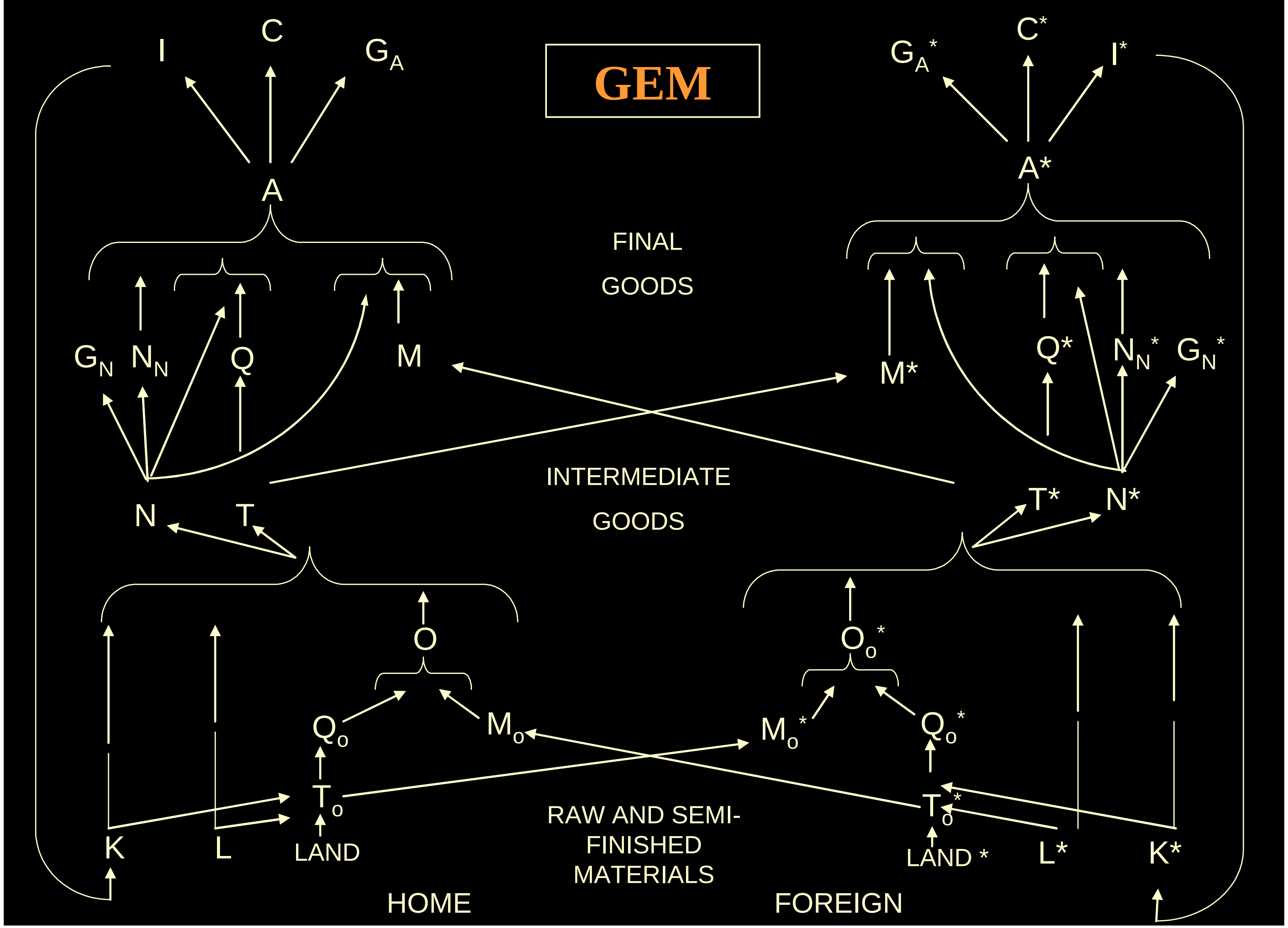
1. Choose a *basic* Newton Algorithm and understand what its doing on your computer!

2. Parameterize the nasty function initially so that it will be easier to solve (where you can either guess the solution or where it will be easier to solve with bad guesses for the endogenous variables).

3. Once you have a solution then as long as the functions are continuous and differentiable Newton will get you to the desired solution as long as you take small enough steps. And it will get you there very quickly!

4. If the function is not continuos or differentiable approximate it with a function that it is. Classic example is a kink (or the zero interest rate floor) which can be approximated with a continuous and differential function like the arctan function.

Figure 5: The Global Economy Model (GEM)



Understanding Newton Methods Using a Simple 2 Equation Linear Example

$$
\begin{aligned}
\mathsf{Y} &= X \\
Y &= \mathsf{2} - X
\end{aligned}
$$

What is Newton's Method?

Rewrite the model in the form

$$\mathsf{F}(\mathsf{Y},\mathsf{Z},\theta) = 0$$

example

$$\begin{aligned} X - Y &= 0 \\ -X - Y - 2 &= 0 \end{aligned}$$

Figure 1: Example of a Simple Linear Model

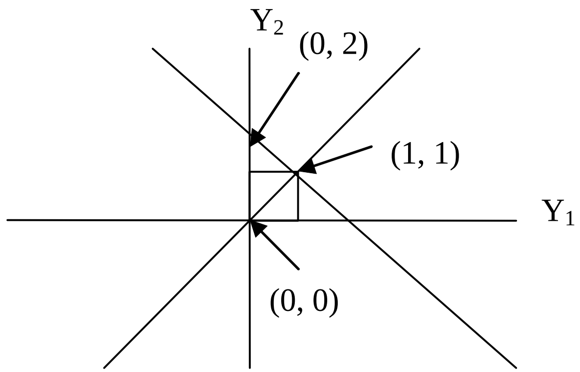


Figure 2:

Find the Jacobian which is simply the matrix of partial derivatives

$$
\begin{aligned}
-X + Y &= 0 \\
X + Y - 2 &= 0
\end{aligned}
$$

$$
[J] = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}
$$

For any given values of X and Y define the residuals of each equation. For example, if we guessed X and Y to be both zero the residuals would be

$$
\begin{aligned}
-X + Y &= 0 \\
X + Y - 2 &= 0
\end{aligned}
$$

$$
\begin{array}{c} RES1 \\ RES2 \end{array} = \begin{array}{c} 0 - 0 = 0 \\ -0 - 0 - 2 = -2 \end{array}
$$

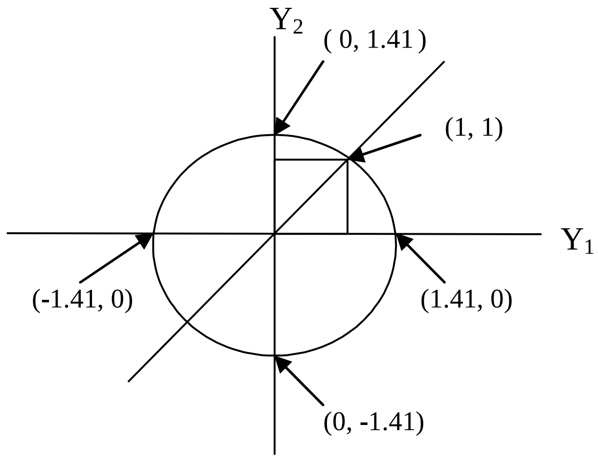
Take a newton step which is simply

$$\begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = -[J]^{-1} \begin{bmatrix} RES1 \\ RES2 \end{bmatrix} = -\begin{bmatrix} .5 & -.5 \\ -.5 & -.5 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

It may be time for a complicated nonlinear problem...

$$\begin{aligned} -X + Y &= 0 \\ X^2 - Y^2 - 2 &= 0 \end{aligned}$$

Figure 3: Example of a Simple Nonlinear Model



Find the Jacobian which is simply the matrix of partial derivatives

$$
\begin{aligned}
X - Y &= 0 \\
X^2 - Y^2 - 2 &= 0
\end{aligned}
$$

$$
[J] = \begin{bmatrix} -1 & 1 \\ 2X & 2Y \end{bmatrix}
$$

Figure 4: Fantastic! Now we are ready to solve GIMF.